Question 1

A *Bungee Jumper*

(a) The jumper comes to rest when

lost gravitational potential energy = stored strain energy

$$mgy = \tfrac{1}{2}k(y-L)^2$$

0.1

$$ky^2 - 2y(kL + mg) + kL^2 = 0$$

0.1

This is solved as a quadratic.

$$y = \frac{2(kL+mg) \pm \sqrt{4(kL+mg)^2 - 4k^2L^2}}{2k}$$

$$= \frac{kL + mg \pm \sqrt{2mgkL + m^2g^2}}{k}$$

0.2

Need positive root; lower position of rest (other root after initial rise).

0.1 ______ 0.5

(b) The maximum speed is attained when the acceleration is zero and forces balance; i.e. when $mg = kx$ 0.1

Also kinetic energy = lost potential energy – strain energy within elastic rope

0.1

$$\tfrac{1}{2}mv^2 = mg(L+x) - \tfrac{1}{2}kx^2$$

0.1

$$x = \frac{mg}{k}$$

$$v^2 = 2g\left(L + \frac{mg}{k}\right) - \frac{mg^2}{k}$$

$$v = \sqrt{2gL + \frac{mg^2}{k}}$$

0.2 ______ 0.5

(c) Time to come to rest = time in free fall + time in SHM of rope to stop stretching

0.1

Length of free fall = $L = \tfrac{1}{2}gt_f^{\,2}$

Therefore $t_f = \sqrt{\frac{2L}{g}}$

0.2

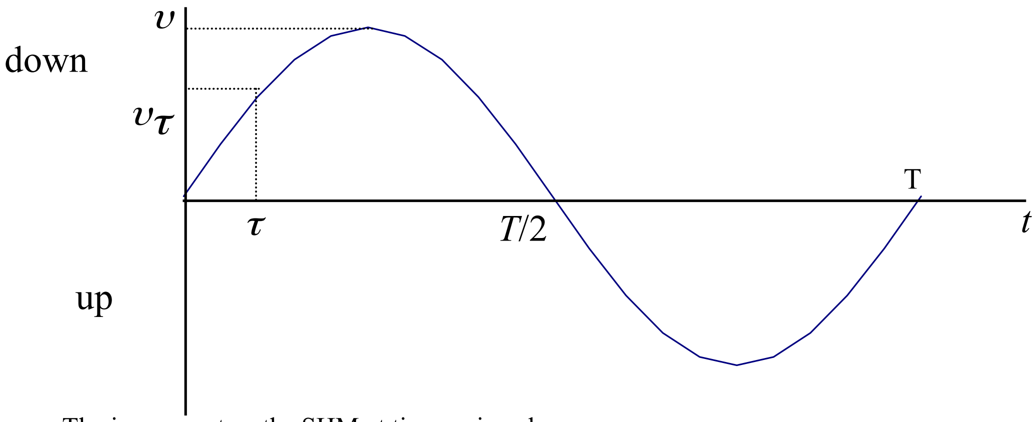
The jumper enters the SHM with free fall velocity = $gt_f = \sqrt{2gL} = v_\tau$

0.1

$$\text{Period of SHM} = 2\pi\sqrt{\frac{m}{k}} = T$$

0.1

We represent a full SHM cycle by



The jumper enters the SHM at time τ given by

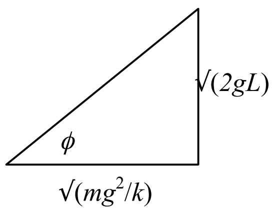
$$\tau = \frac{1}{\omega}\sin^{-1}\frac{\upsilon_\tau}{\upsilon} = \frac{1}{\omega}\sin^{-1}\frac{\sqrt{2gL}}{\upsilon}$$

0.2

Jumper comes to rest at one half cycle of the SHM at total time given by

$= t_f + (T/2 \ - \tau)$

0.1



$$= \sqrt{\frac{2L}{g}} + \pi\sqrt{\frac{m}{k}} - \frac{1}{\omega}\sin^{-1}\frac{\sqrt{2gL}}{\upsilon}$$

$$= \sqrt{\frac{2L}{g}} + \pi\sqrt{\frac{m}{k}} - \frac{1}{\omega}\sin^{-1}\frac{\sqrt{2gL}}{\sqrt{2gL + mg^2/k}}$$

$$= \sqrt{\frac{2L}{g}} + \sqrt{\frac{m}{k}}\ \{\pi - \sin^{-1}\frac{\sqrt{2gL}}{\sqrt{2gL + mg^2/k}}\}$$

This is the same as

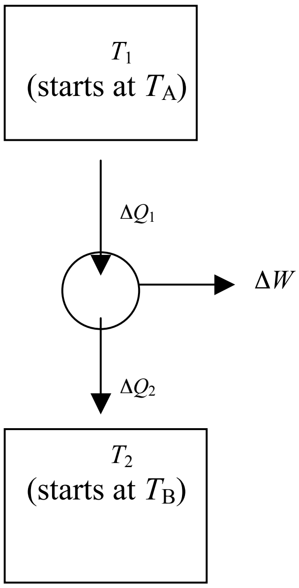
$$= \sqrt{\frac{2L}{g}} + \sqrt{\frac{m}{k}}\ \{\frac{\pi}{2} + \cos^{-1}\frac{\sqrt{2gL}}{\sqrt{2gL + mg^2/k}}\}$$

$$= \sqrt{\frac{2L}{g}} + \sqrt{\frac{m}{k}}\tan^{-1}\{-\sqrt{\frac{2kL}{mg}}\}$$

0.2______

1.0

B Heat Engine Question



In calculating work obtainable,
we assume no loss (friction etc.) in engine working.

ΔQ_1 = energy from body A
$= -ms\Delta T_1$ (ΔT_1 –ve)

$\Delta Q_2 = ms\Delta T_2$ (ΔT_2 +ve)

(a) For maximum amount of mechanical energy assume Carnot engine

$$\frac{\Delta Q_1}{T_1} = \frac{\Delta Q_2}{T_2} \quad \text{throughout operation (second law)}$$

0.2

But $\Delta Q_1 = -ms\Delta T_1$ and $\Delta Q_2 = ms\Delta T_2$

0.2

$$-ms\int_{T_A}^{T_0}\frac{dT_1}{T_1} = ms\int_{T_B}^{T_0}\frac{dT_2}{T_2}$$

0.1

$$\ln\frac{T_A}{T_0} = \ln\frac{T_0}{T_B}$$

0.1

$$T_0^2 = T_A T_B$$
$$T_0 = \sqrt{T_A T_B}$$

0.2

0.8

$$Q_1 = -ms \int_{T_A}^{T_0} \mathrm{d}T_1 = ms(T_A - T_0)$$

0.2

$$Q_2 = ms \int_{T_B}^{T_0} \mathrm{d}T_2 = ms(T_0 - T_B)$$

0.1

$$W = Q_1 - Q_2$$

0.2

$$W = ms(T_A - T_0 - T_0 + T_B) = ms(T_A + T_B - 2T_0) = ms\,(T_A + T_B - 2\sqrt{T_A T_B}\,)$$

or $ms\,(\sqrt{T_A} - \sqrt{T_B})^2$

0.2
0.7

(d) Numerical example:

Mass = volume × density

$W = 2.50 \times 1.00 \times 10^3 \times 4.19 \times 10^3 \times (350 + 300 - 2\sqrt{350 \times 300}\,)$ J

$= 20 \times 10^6$ J

$=$ 20 MJ

0.5

0.5

C Radioactivity and age of the Earth

(a) $N = N_0 e^{-\lambda}$ N_0 = original number

0.1

$n = N_0(1 - e^{-\lambda t})$

0.1

Therefore $n = N e^{\lambda t}(1 - e^{-\lambda t}) = N(e^{\lambda t} - 1)$

0.1

So $n = N(2^{t/\tau} - 1)$ where τ is half-life

or as $\lambda = \frac{\ln 2}{T} = \frac{0.6931}{T}$, n = N($e^{\frac{0.6931t}{T}} - 1$)

0.1

${}^{206}n = {}^{238}N(2^{t/4.50} - 1)$ or ${}^{206}n = {}^{238}N(e^{0.1540t} - 1)$ where time t is in 10^9 years

0.1

0.5

(b) ${}^{207}n = {}^{235}N(2^{t/0.710} - 1)$ or ${}^{207}n = {}^{235}N(e^{0.9762t} - 1)$

0.1

0.1

(c) In mixed uranium (i.e. containing Pb of both natural and radioactive origin)

204 : 206 : 207 have proportions	1.00 : 29.6 : 22.6
In pure lead (no radioactivity)	1.00 : 17.9 : 15.5

Therefore for radioactively produced lead by subtraction

2
0
6

:

2
0
7

1
1
.
7

:

7
.
1

0.2

Dividing equations from (a) and (b) gives

$$\frac{^{206}n}{^{207}n} = \frac{^{238}N}{^{235}N}\{\frac{2^{t/4.50}-1}{2^{t/0.710}-1}\} \text{ or } \frac{^{206}n}{^{207}n} = \frac{^{238}N}{^{235}N}\{\frac{e^{0.1540t}-1}{e^{0.9762t}-1}\}$$

0.1

$$\frac{11.7}{7.1} = 137\{\frac{2^{T/4.50}-1}{2^{T/0.710}-1}\} \text{ or } \frac{11.7}{7.1} = 137\{\frac{e^{0.1540T}-1}{e^{0.9762T}-1}\}$$

0.1

$$0.0120\ \{2^{T/0.710}-1\} = \{2^{T/4.50}-1\}$$

$$\text{or } 0.0120\ \{e^{0.9762T}-1\} = \{e^{0.1540T}-1\}$$

0.1

0.5

(d) Assume $T >> 4.50 \times 10^9$ and ignore 1 in both brackets: 0.2

$$0.0120\ \{2^{T/0.710}\} = \{2^{T/4.50}\} \text{ or } 0.0120\ \{e^{0.9762T}\} = \{e^{0.1540T}\}$$

$$0.0120 = \{2^{T/4.50\ -\ T/0.710}\} = 2^{T(0.222-1.4084)} = 2^{-1.1862T}$$

$$T = -\frac{\log 0.0120}{\log 2 \times 1.1862} = 5.38$$

$T = 5.38 \times 10^9$ years

$$\text{or } 0.0120 = e^{-0.8222T} \qquad T = \frac{\ln 0.0120}{-0.8222} = \frac{-4.4228}{-0.8222} = 5.38$$

$T = 5.38 \times 10^9$ years

0.2

0.4

(e) T is not >> 4.50×10^9 years but is > 0.71×10^9 years 0.1

We can insert the approximate value for T (call it $T^* = 5.38 \times 10^9$ years) in the $2^{T/4.50}$ term and obtain a better value by iteration in the rapidly changing $2^{T/0.710}$ term). We now leave in the –1's, although the –1 on the right-hand side has little effect and may be omitted). 0.1

Either $$0.0120((2^{T/0.710} - 1) = 2^{T^*/4.50} - 1$$

$$2^{T/0.710} - 1 = \frac{2^{1.1956} - 1}{0.0120} = \frac{2.2904 - 1}{0.0120} = 107.5$$

$$T = 0.710\frac{\log 108.5}{\log 2} = 4.80(0)$$

0.2

Put* T* = *4.80(0)* $\times$ *10^9 years

$$2^{T/0.710} = \frac{2^{1.0668} - 1}{0.0120} = \frac{2.0948 - 1}{0.0120} = 91.2$$

$$T = 0.710\frac{\log 91.2}{\log 2} = 4.62(3)$$

Further iteration gives 4.52

0.1

or

$0.0120(e^{0.9762T} - 1) = (e^{0.1540T^*} - 1)$ and similar

So more accurate answer for* T *to be in range 4.6 $\times$ 10^9 years to 4.5 $\times$ 10^9 years (either acceptable).

0.5

D Spherical charge

(a) Charge density = $\rho = \dfrac{Q}{\frac{4}{3}\pi R^3}$ within sphere

0.3

$x \leq R$ Field at distance x:

$$E = \frac{\frac{4}{3}\pi x^3 \rho}{4\pi\varepsilon_0 x^2} = \frac{Qx}{4\pi\varepsilon_0 R^3}$$

0.3

$x > R$ Field at distance x from the centre: $E = \dfrac{Q}{4\pi\varepsilon_0 x^2}$

0.2
0.8

(b) Method 1

Energy density is $\frac{1}{2}\varepsilon_0 E^2$.

0.1

$x \leq R$

Energy in a thin shell of thickness δx at radius x is given by

$$= \frac{1}{2}\varepsilon_0 E^2 \; 4\pi x^2 \delta x = \frac{1}{2} 4\pi\varepsilon_0 \frac{Q^2 x^2}{(4\pi\varepsilon_0)^2 R^6} x^2 \,\delta x$$ 0.1

Energy within the spherical volume = $\dfrac{1}{2}\dfrac{Q^2}{(4\pi\varepsilon_0)R^6}\displaystyle\int_{x=0}^{x=R} x^4 \mathrm{d}x = \dfrac{1}{40}\dfrac{Q^2}{\pi\varepsilon_0}\dfrac{1}{R}$

0.2

$x > R$

Energy within spherical shell = $\frac{1}{2}\varepsilon_0 E^2 \; 4\pi x^2 \delta x = \frac{1}{2} 4\pi\varepsilon_0 \dfrac{Q^2}{(4\pi\varepsilon_0)^2 x^4} x^2 \,\delta x$

0.1

Energy within the spherical volume for $x > R$

$$= \frac{1}{2}\frac{Q^2}{(4\pi\varepsilon_0)}\int_{x=R}^{x=\infty} \frac{1}{x^2}\mathrm{d}x = \frac{1}{8}\frac{Q^2}{\pi\varepsilon_0}\frac{1}{R}$$

0.2

Total energy associated with the charge distribution = $\dfrac{1}{40}\dfrac{Q^2}{\pi\varepsilon_0}\dfrac{1}{R}$

$+ \dfrac{1}{8}\dfrac{Q^2}{\pi\varepsilon_0}\dfrac{1}{R}$

$$= \frac{3}{20}\frac{Q^2}{\pi\varepsilon_0}\frac{1}{R}$$

0.1
0.8

Method 2

A shell with charge $4\pi x^2 \delta x \rho$ moves from ∞ to the surface of a sphere radius x

0.1

where the electric potential is

$$\frac{\frac{4}{3}\pi x^3 \rho}{4\pi\varepsilon_0 x} = \frac{x^2 \rho}{3\varepsilon_0}$$

0.2

and will therefore gain electrical potential energy $\left(\frac{x^2 \rho}{3\varepsilon_0}\right)\left(4\pi r^2 \rho\right)\delta x$

0.1

Total energy of complete sphere = $\int_{x=0}^{x=R} \frac{4\pi\rho^2 x^4}{3\varepsilon_0} dx = \frac{4}{15}\frac{\pi\rho^2 R^5}{\varepsilon_0}$

0.2

Putting Q = charge on sphere = $\frac{4}{3}\pi R^3 \rho$, $\rho = \frac{3Q}{4\pi R^3}$

So that total energy is = $\frac{4}{15}\pi\left(\frac{9Q^2}{16\pi^2 R^6}\right)\frac{R^5}{\varepsilon} = \frac{3}{20}\frac{Q^2}{\pi\varepsilon_0 R}$

0.2

0.8

(c) Binding energy $E_{\text{binding}} = E_{\text{electric}} - E_{\text{nuclear}}$

0.1

Binding energy is a negative energy

Therefore -8.768 = E_{electric} - 10.980 MeV per nucleon

E_{electric} = 2.212 MeV per nucleon

0.1

Radius of cobalt nucleus is given by $R = \frac{3}{20}\frac{Q^2}{\pi\varepsilon_0 E^{total}_{electric}}$

$$= \frac{3 \times 27^2 \times (1.60 \times 10^{-19})^2}{20 \times \pi \times 8.85 \times 10^{-12} \times 2.212 \times 10^6 \times 57 \times 1.60 \times 10^{-19}} \text{ m}$$

= 5.0×10^{-15} m

0.2

0.4

E E.M. Induction

Method 1 Equating energy

Horizontal component of magnetic field B inducing emf in ring:

$B = 44.5 \times 10^{-6} \cos 64^{o}$ 0.2

Magnetic flux through ring at angle $\theta = B\pi a^2 \sin\theta$

where a = radius of ring 0.1

Instantaneous emf = $\dfrac{d\phi}{dt} = B\pi a^2 \dfrac{d \sin \omega t}{dt}$ where ω = angular velocity

$= B\pi a^2 \omega \cos \omega t = B\pi a^2 \omega \cos \theta$ 0.1

R.m.s. emf over 1 revolution = $\dfrac{B\pi a^2 \omega}{\sqrt{2}}$ ***0.2***

Average resistive heating of ring = $\dfrac{B^2 \pi^2 a^4 \omega^2}{2R}$ 0.1

Moment of inertia = $\dfrac{1}{2} m a^2$ 0.1

Rotational energy = $\dfrac{1}{4} m a^2 \omega^2$ where m = mass of ring 0.1

Power producing change in ω = $\dfrac{d}{dt}\left\{\dfrac{1}{4} m a^2 \omega^2\right\} =$

$\dfrac{1}{4} m a^2 2\omega \dfrac{d\omega}{dt}$

0.1

Equating: $$\frac{1}{2} m a^2 \omega \frac{d\omega}{dt} = -\frac{B^2 \pi^2 a^4 \omega^2}{2R}$$

0.1

$$\frac{d\omega}{\omega} = -\frac{B^2 \pi^2 a^2}{mR} dt$$

0.1

If T is time for angular velocity to halve,

$$\int_{\omega}^{\omega/2} \frac{d\omega}{\omega} = -\int_0^T \frac{B^2 \pi^2 a^2}{mR} dt$$

0.1

$$\ln 2 = \frac{B^2 \pi^2 a^2}{mR} T$$ 0.2

But R = $\dfrac{2\pi a\rho}{A}$ where A is cross-sectional area of copper ring 0.1

$m = 2\pi a d A$ (d = density) 0.1

$$\ln 2 = \frac{B^2 \pi^2 a^2 T}{\dfrac{2\pi a\rho}{A} 2\pi a d A} = \frac{B^2 T}{4\rho d}$$ 0.1

$$T = \frac{4\rho d \ln 2}{B^2} = \frac{4 \times 1.70 \times 10^{-8} \times 8.90 \times 10^{3} \times 0.6931}{(44.5 \times 10^{-6} \times 0.4384)^2} \text{s}$$

$= 1.10(2) \times 10^6$ s (=306 hr = 12 days 18 hr) 0.2

2.0

(Part E)

Method 2 Back Torque

Horizontal component of magnetic field = B = $44.5 \times 10^{-6} \cos 64^{o}$ 0.2

Cross-section of area of ring is A

Radius of ring = a

Density of ring = d

Resistivity = ρ

ω = angular velocity (ω positive when clockwise)

Resistance $R = \rho \dfrac{2\pi a}{A}$ 0.1

Mass of ring m = $2\pi a A d$ 0.1

Moment of inertia = $M = \dfrac{1}{2} m a^2$ 0.1

Magnetic flux through ring at angle $\theta = B\pi a^2 \sin\theta$ 0.1

Instantaneous emf = $\dfrac{d\phi}{dt} = B\pi a^2 \dfrac{d \sin \omega t}{dt} = B\pi a^2 \omega \cos \omega t = B\pi a^2 \omega \cos\theta$ 0.1

Induced current = I = $B\pi a^2 \cos\theta / R$

Torque opposing motion = $(B\pi a^2 \cos\theta)\, I = \dfrac{1}{R}\left(B\pi\, a^2\right)^2 \omega \cos^2\theta$ 0.1

Work done in small $\delta\theta = \dfrac{1}{R}\left(B\pi\, a^2\right)^2 \omega \dfrac{1}{2}\left(\cos 2\theta + 1\right)\delta\theta$ 0.1

Average torque = (work done in 2π revolution)/2π

$$= \frac{1}{2\pi\, R}\left(B\pi\pi^{\;2}\right)^2 \omega \frac{1}{2} 2\pi = \frac{1}{2R}\left(B\pi\pi^{\;2}\right)^2 \omega$$ 0.1

This equals $M\dfrac{d\omega}{dt}$ so that $M\dfrac{d\omega}{dt} = -\dfrac{B(\pi a^2)B(\pi a^2)\frac{1}{2}}{(\rho / A)(2\pi a)}\omega$ 0.2

$$\frac{1}{2}(2\pi a A d)a^2 \frac{d\omega}{dt} = -\frac{B^2(\pi a^2)^2 A}{4\rho\pi a}\omega$$

$$\frac{d\omega}{dt} = -\frac{B^2}{4\rho d}\omega$$ 0.2

$$\int_{\omega}^{\omega/2} \frac{d\omega}{\omega} = \int_{0}^{T} \frac{B^2}{4\rho d} dt$$ 0.2

$$\ln 2 = \frac{B^2 T}{4\rho d}$$ 0.2

$$T = \frac{4\rho d \ln 2}{B^2} = \frac{4 \times 1.70 \times 10^{-8} \times 8.90 \times 10^3 \times 0.6931}{(44.5 \times 10^{-6} \times 0.4384)^2}\text{s}$$

$= 1.10(2) \times 10^6$ s =306 hr = 12 days 18 hr

0.2

2.0

Question Two ~ Solution

(a) Focusing occurs for one "cyclotron" orbit of the electron.

Angular velocity $\omega = \mathrm{e}\, B / \mathrm{m}$; so time for one orbit $T = 2\, \pi \mathrm{m} / \mathrm{e}\, B$

Speed of electron $u = (2\, \mathrm{e}\, V / \mathrm{m})^{1/2}$

Distance travelled $D = T\, u \cos \beta \approx T\, u = (2^{3/2}\, \pi / B)\, (V \mathrm{m} / \mathrm{e})^{1/2}$

Thus charge to mass ratio = $\mathrm{e} / \mathrm{m} = 8\, \mathrm{V} \times (\pi / B\, D)^2$

(b) Consider condition (ii) - Force due to electric field acts upwards

In region A force due magnetic field acts upwards as well, electron hits upper plate and does not reach the film.

In region B, force due magnetic field acts downwards, and *if* force is equal and opposite to the electrostatic force, there will be no unbalanced force, and electron will emerge from plates to expose film.

Piece was taken from region B.

(c) We require forces to balance. Electric force given by $\mathrm{e}V / t$, magnitude of magnetic force given by $\mathrm{e}\, u\, B \sin \phi$, with u the speed of the electron.

For these to balance we require $u = V / t\, B\, |\sin \phi|$

Maximum u corresponds to minimum ϕ - at 23°

Therefore $u = 2.687 \times 10^8$ m/s = 0.896 c.

Relativistic $\gamma = (1 - v^2/\mathrm{c}^2)^{-1/2} = 2.255$,
so kinetic energy of electron = $(\gamma\text{-}1)\, \mathrm{m}\, \mathrm{c}^2$ = 641 keV.

(d) After emerging from region between plates, electrons experience force due to magnetic field only. We approximate this by a vertical force, because angle of electron to horizontal remains small.

Acceleration caused by this force $a = B\,\mathrm{e}\,u \sin\phi / \gamma\,\mathrm{m}$

Initial horizontal speed is u, therefore time taken to reach the film after emerging from the region between the plates $t = s / u$.

Change in vertical displacement during this time $= y / 2 = ½\, a\, (s / u)^2$

$y = B\,\mathrm{e}\,s^2 \sin\phi / \gamma\,\mathrm{m}\,u$

From part (f), for electron to have emerged from plate,
we also know $u = V / t\,B\,|\sin\phi|$.

Therefore we eliminate u to obtain:

$$y^2 = (\mathrm{e}\,B\,s \sin\phi / \mathrm{m})^2 \left\{(B\,s\,t \sin\phi / V)^2 - (s / \mathrm{c})^2\right\}$$

and we plot VERTICAL $(y / B\,s \sin\phi)^2$

HORIZONTAL $(B\,s\,t \sin\phi / V)^2$

Therefore we have a gradient $(\mathrm{e} / \mathrm{m})^2$

and a vertical-axis intercept $-(\mathrm{e}\,s / \mathrm{m}\,\mathrm{c})^2$

The intercept is read as -537.7 $(\mathrm{C\,s / kg})^2$, giving e/m = 1.70×10^{11} C / kg

The gradient is read as 2.826×10^{22} $(\mathrm{C/kg})^2$, giving e/m = 1.68×10^{11} C / kg.

MARK SCHEME AND SOLUTIONS FOR Q3

Total marks = 10

A	**a)**	$\Delta x_t = ae^{-\mu t}\cos(\omega t + \phi)$, $0.8 = e^{-50\mu} \Rightarrow \mu = 4.5 \times 10^{-3}$ s^{-1}.	[0.1]
	b)	$\upsilon = (E/\rho)^{½} = (7.1 \times 10^{10}/2700)^{½} = 5100$ m.s^{-1}.	
		At fundamental $\lambda_{rod} = 4l = 4$ m.	
		$f = 5100 / 4 = 1.3 \times 10^3$ Hz.	
		$\omega = 2\pi f = 8.1 \times 10^3$ rad.s^{-1}.	[0.1]
	c)	$\upsilon = f\lambda_{rod}$, $\delta\lambda_{rod} / \lambda_{rod} = (-)\delta f / f \Rightarrow \delta l / l.$	[0.8]
		$\delta l = l.(\delta f / f)$.	[0.6]
		$\delta l = 1 \times (5.0 \times 10^{-3}/1.3 \times 10^3) = 3.8 \times 10^{-6}$ m.	[0.1]
	d)	Change in gravitational force on rod at a distance x from the free end = $m\Delta g$ and $m = \rho x A$,	
		where A is the cross-sectional area of the rod.	[0.5]
		Change in stress = $m\Delta g/A = \rho x \Delta g$.	[0.5]
		Change in strain = $\delta(dx)/dx = \rho x \Delta g/E$;	
		that is, $dx \rightarrow (1+ \rho x \Delta g/E)dx \Rightarrow \Delta l = (\rho \Delta g/2E)l^2$.	[0.5]
	e)	At fundamental $\lambda_{rod} = 4l \Rightarrow \Delta l = \Delta\lambda_{rod}/4$,	
		for $\Delta\lambda_{rod} = 656$ nm$/10^4 \Rightarrow \Delta l = 656$ nm $/(4 \times 10^4)$.	[0.1]
		$\Delta l = 656$ nm $/(4 \times 10^4) = (\rho\Delta g/2E)l^2$	[0.1]
		$\Delta l = (2700 \times 10^{-19}/ 14 \times 10^{10})\, l^2 \Rightarrow l = 9.2 \times 10^7$ m.	[0.1]
B	**a)**	$mc^2 = hf \Rightarrow m = hf/c^2$,	[0.3]
		$hf' = hf - GMm/R$,	[0.3]
		$\Rightarrow hf' = hf(1 - GM/Rc^2)$, $\therefore f' = f(1 - GM/Rc^2)$.	[0.4]
	b)	$n_r = c / c(1 - GM/rc^2)^2$,	[1.0]
		$n_r = 1 + 2GM/rc^2$, for small GM/rc^2; i.e. $\alpha = 2$.	[1.0]

c)

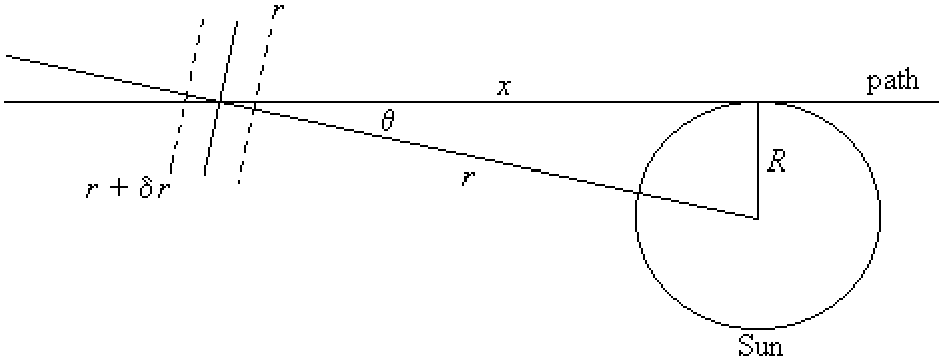


Diagram [0.2]

By Snell's law: $n(r + \delta r)\sin\theta = n(r)\sin(\theta - \delta\xi)$, [1.0]

$(n(r) + (dn/dr)\,\delta r)\sin\theta = n(r)\sin\theta - n(r)\cos\theta\,\delta\xi$. [0.4]

$(dn/dr)\,\delta r\sin\theta = -\,n(r)\cos\theta\,\delta\xi$.

Now $n(r) = 1 + 2GM/rc^2$, so $(dn/dr) = -\,2GM/c^2r^2$, [0.3]

and $(2GM/c^2r^2)\sin\theta\,\delta r = n(r)\cos\theta\,\delta\xi$.

Hence $\delta\xi = (2GM/c^2r^2)\tan\theta\,(\delta r/n) \approx (2GM\tan\theta/c^2r^2)\delta r$. [1.0]

Now $r^2 = x^2 + R^2$, so $rdr = xdx$. [0.1]

$$\int d\xi = \frac{2GM}{c^2}\int\frac{\tan\theta dr}{r^2} = \frac{2GM}{c^2}\int\frac{\tan\theta r dr}{r^3} = \frac{2GMR}{c^2}\int_{-\infty}^{+\infty}\frac{dx}{(x^2+R^2)^{3/2}}$$

$$\xi = \frac{4GM}{Rc^2}\,radians = 8.4\times10^{-6}\,radians.$$

[0.5]