Theory Question 1: Gravity in a Neutron Interferometer

Enter all your answers into the **Answer Script**.

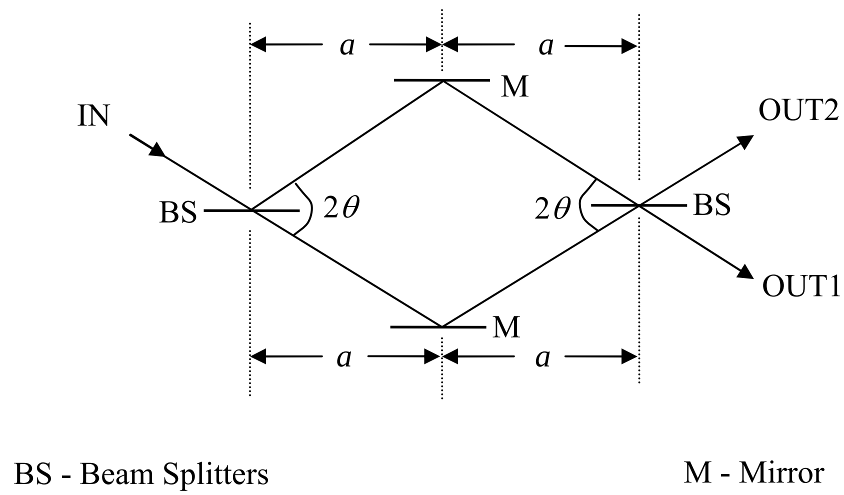

Figure 1a

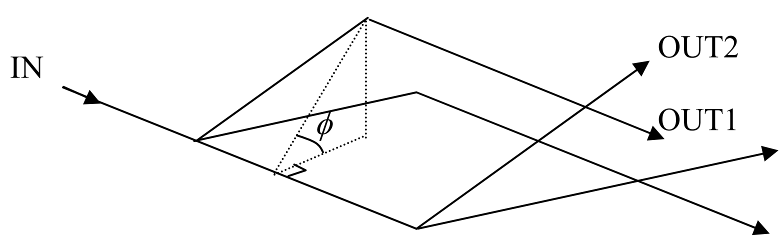

Figure 1b

Physical situation We consider the situation of the famous neutron-interferometer experiment by Collela, Overhauser and Werner, but idealize the set-up inasmuch as we shall assume perfect beam splitters and mirrors within the interferometer. The experiment studies the effect of the gravitational pull on the de Broglie waves of neutrons.

The symbolic representation of this interferometer in analogy to an optical interferometer is shown in Figure 1a. The neutrons enter the interferometer through the IN port and follow the two paths shown. The neutrons are detected at either one of the two output ports, OUT1 or OUT2. The two paths enclose a diamond-shaped area, which is typically a few cm^2 in size.

The neutron de Broglie waves (of typical wavelength of 10^{-10} m) interfere such that all neutrons emerge from the output port OUT1 if the interferometer plane is horizontal. But when the interferometer is tilted around the axis of the incoming neutron beam by angle ϕ (Figure 1b), one observes a ϕ dependent redistribution of the neutrons between the two output ports OUT1 and OUT2.

Geometry For $\phi = 0°$ the interferometer plane is horizontal; for $\phi = 90°$ the plane is vertical with the output ports above the tilt axis.

1.1 (1.0) How large is the diamond-shaped area A enclosed by the two paths of the interferometer?

1.2 (1.0) What is the height H of output port OUT1 above the horizontal plane of the tilt axis?

Express A and H in terms of a, θ, and ϕ.

Optical path length The optical path length N_{opt} (a number) is the ratio of the geometrical path length (a distance) and the wavelength λ. If λ changes along the path, N_{opt} is obtained by integrating λ^{-1} along the path.

1.3 (3.0) What is the difference ΔN_{opt} in the optical path lengths of the two paths when the interferometer has been tilted by angle ϕ? Express your answer in terms of a, θ, and ϕ as well as the neutron mass M, the de Broglie wavelength λ_0 of the incoming neutrons, the gravitational acceleration g, and Planck's constant h.

1.4 (1.0) Introduce the volume parameter

$$V = \frac{h^2}{gM^2}$$

and express ΔN_{opt} solely in terms of A, V, λ_0, and ϕ. State the value of V for $M = 1.675 \times 10^{-27}\,\text{kg}$, $g = 9.800\,\text{m}\,\text{s}^{-2}$, and $h = 6.626 \times 10^{-34}\,\text{J}\,\text{s}$.

1.5 (2.0) How many cycles — from high intensity to low intensity and back to high intensity — are completed by output port OUT1 when ϕ is increased from $\phi = -90°$ to $\phi = 90°$?

Experimental data The interferometer of an actual experiment was characterized by $a = 3.600\,\text{cm}$ and $\theta = 22.10°$, and 19.00 full cycles were observed.

1.6 (1.0) How large was λ_0 in this experiment?

1.7 (1.0) If one observed 30.00 full cycles in another experiment of the same kind that uses neutrons with $\lambda_0 = 0.2000\,\text{nm}$, how large would be the area A?

Hint: If $|\alpha x| << 1$, it is permissible to replace $(1+x)^{\alpha}$ by $1+\alpha x$.

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Answer Script

Geometry

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1.1 The area is

$A =$

1.0

1.2 The height is

$H =$

1.0

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Optical path length

1.3 In terms of a, θ, ϕ, M, λ_0, g, and h:

$\Delta N_{\text{opt}} =$

1.4 In terms of A, V, λ_0, and ϕ:

$\Delta N_{\text{opt}} =$

The numerical value of V is

$V =$

1.5 The number of cycles is

# of cycles =

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3.0

0.8

0.2

2.0

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Experimental data

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1.6 The de Broglie wavelength was

$\lambda_0 =$

1.0

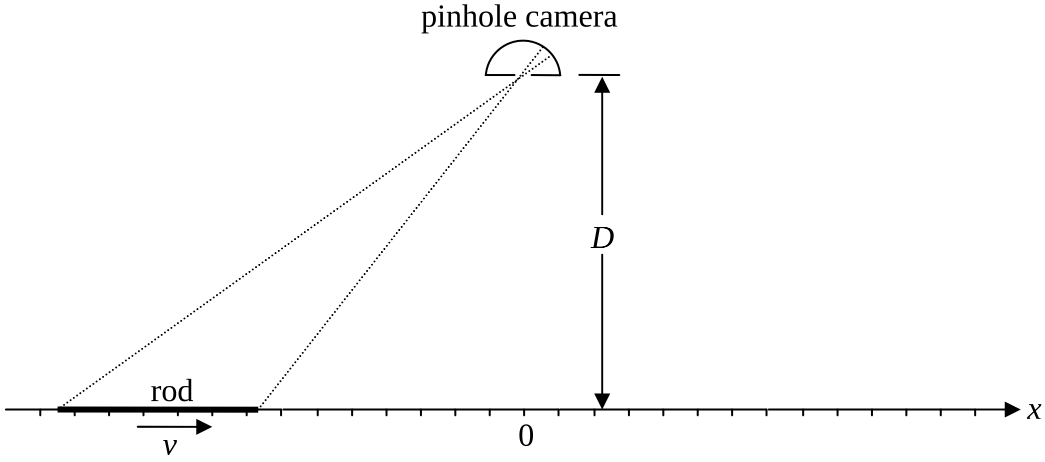
1.7 The area is

$A =$

1.0

Theory Question 2: Watching a Rod in Motion

Enter all your answers into the **Answer Script**.



Physical situation A pinhole camera, with the pinhole at $x = 0$ and at distance D from the x axis, takes pictures of a rod, by opening the pinhole for a very short time. There are equidistant marks along the x axis by which the *apparent length* of the rod, as it is seen on the picture, can be determined from the pictures taken by the pinhole camera. On a picture of the rod *at rest*, its length is L. However, the rod is *not* at rest, but is moving with constant velocity υ along the x axis.

Basic relations A picture taken by the pinhole camera shows a tiny segment of the rod at position $\tilde{x}$.

2.1 (0.6) What is the *actual position* x of this segment at the time when the picture is taken? State your answer in terms of $\tilde{x}$, D, L, υ, and the speed of light $c = 3.00\times10^8\,\mathrm{m\,s^{-1}}$. Employ the quantities

$$\beta = \frac{\upsilon}{c} \quad \text{and} \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

if they help to simplify your result.

2.2 (0.9) Find also the corresponding inverse relation, that is: express $\tilde{x}$ in terms of x, D, L, υ, and c.

Note: The *actual position* is the position in the frame in which the camera is at rest

Apparent length of the rod The pinhole camera takes a picture at the instant when the actual position of the center of the rod is at some point x_0.

2.3 (1.5) In terms of the given variables, determine the apparent length of the rod on this picture.

2.4 (1.5) Check one of the boxes in the **Answer Script** to indicate how the apparent length changes with time.

Symmetric picture One pinhole-camera picture shows both ends of the rod at the same distance from the pinhole.

2.5 (0.8) Determine the apparent length of the rod on this picture.

2.6 (1.0) What is the actual position of the middle of the rod at the time when this picture is taken?

2.7 (1.2) Where does the picture show the image of the middle of the rod?

Very early and very late pictures The pinhole camera took one picture very early, when the rod was very far away and approaching, and takes another picture very late, when the rod is very far away and receding. On one of the pictures the apparent length is 1.00 m, on the other picture it is 3.00 m.

2.8 (0.5) Check the box in the **Answer Script** to indicate which length is seen on which picture.

2.9 (1.0) Determine the velocity υ.

2.10 (0.6) Determine the length L of the rod at rest.

2.11 (0.4) Infer the apparent length on the symmetric picture.

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Answer Script

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Basic Relations

2.1 x value for given $\tilde{x}$ value:

$x =$

0.6

2.2 $\tilde{x}$ value for given x value:

$\tilde{x} =$

0.9

Apparent length of the rod

2.3 The apparent length is

$\tilde{L}(x_0) =$

1.5

2.4 Check one: The apparent length

- ☐ increases first, reaches a maximum value, then decreases.
- ☐ decreases first, reaches a minimum value, then increases.
- ☐ decreases all the time.
- ☐ increases all the time.

1.5

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Symmetric picture

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2.5 The apparent length is **0.8**

$\tilde{L} =$

2.6 The actual position of the middle of the rod is **1.0**

$x_0 =$

2.7 The picture shows the middle of the rod at a distance **1.2**

$l =$

from the image of the front end of the rod.

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Very early and very late pictures

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2.8 Check one:

☐ The apparent length is 1 m on the early picture and 3 m on the late picture.

☐ The apparent length is 3 m on the early picture and 1 m on the late picture.

0.5

2.9 The velocity is

$\upsilon =$

1.0

2.10 The rod has length

$L =$

at rest.

0.6

2.11 The apparent length on the symmetric picture is

$\tilde{L} =$

0.4

Theory Question 3

This question consists of five independent parts. Each of them asks for an estimate of an order of magnitude only, not for a precise answer. Enter all your answers into the **Answer Script**.

Digital Camera Consider a digital camera with a square CCD chip with linear dimension L = 35 mm having N_p = 5 Mpix (1 Mpix = 10^6 pixels). The lens of this camera has a focal length of f = 38 mm. The well known sequence of numbers (2, 2.8, 4, 5.6, 8, 11, 16, 22) that appear on the lens refer to the so called F-number, which is denoted by $F\#$ and defined as the ratio of the focal length and the diameter D of the aperture of the lens, $F\# = f / D$.

3.1 (1.0) Find the best possible spatial resolution $\Delta x_{\min}$, at the chip, of the camera as limited by the lens. Express your result in terms of the wavelength λ and the F-number $F\#$ and give the numerical value for λ = 500 nm.

3.2 (0.5) Find the necessary number N of Mpix that the CCD chip should possess in order to match this optimal resolution.

3.3 (0.5) Sometimes, photographers try to use a camera at the smallest practical aperture. Suppose that we now have a camera of N_0 = 16 Mpix, with the chip size and focal length as given above. Which value is to be chosen for $F\#$ such that the image quality is not limited by the optics?

3.4 (0.5) Knowing that the human eye has an approximate angular resolution of ϕ = 2 arcmin and that a typical photo printer will print a minimum of 300 dpi (dots per inch), at what minimal distance z should you hold the printed page from your eyes so that you do not see the individual dots?

<u>Data</u> 1 inch = 25.4 mm
1 arcmin = 2.91×10^{-4} rad

Hard-boiled egg An egg, taken directly from the fridge at temperature $T_0 = 4°C$, is dropped into a pot with water that is kept boiling at temperature T_1.

3.5 (0.5) How large is the amount of energy U that is needed to get the egg coagulated?

3.6 (0.5) How large is the heat flow J that is flowing into the egg?

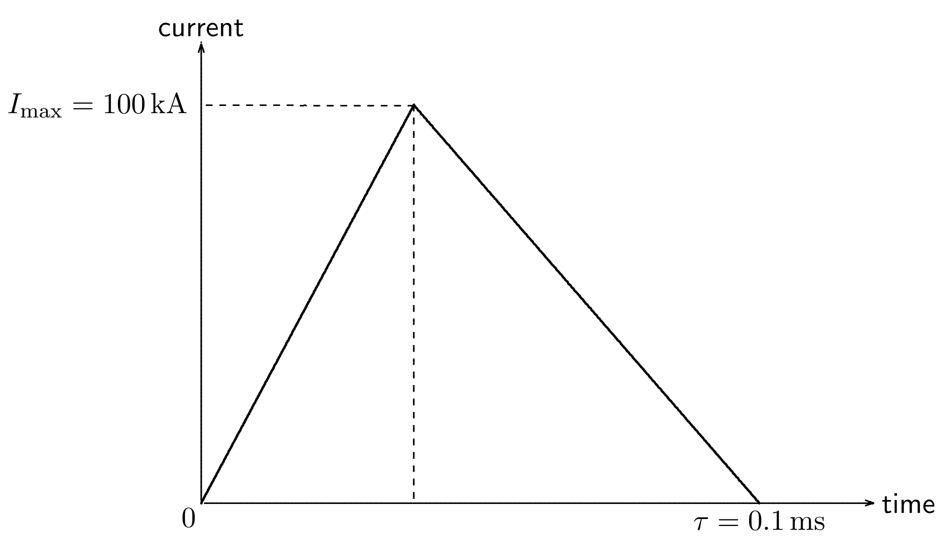
3.7 (0.5) How large is the heat power P transferred to the egg?

3.8 (0.5) For how long do you need to cook the egg so that it is hard-boiled?

Hint You may use the simplified form of Fourier's Law $J = \kappa \Delta T / \Delta r$, where ΔT is the temperature difference associated with Δr, the typical length scale of the problem. The heat flow J is in units of $\mathrm{W\,m^{-2}}$.

Data Mass density of the egg: $\mu = 10^3\,\mathrm{kg\,m^{-3}}$
Specific heat capacity of the egg: $C = 4.2\,\mathrm{J\,K^{-1}\,g^{-1}}$
Radius of the egg: $R = 2.5\,\mathrm{cm}$
Coagulation temperature of albumen (egg protein): $T_c = 65°C$
Heat transport coefficient: $\kappa = 0.64\,\mathrm{W\,K^{-1}\,m^{-1}}$ (assumed to be the same for liquid and solid albumen)

Lightning An oversimplified model of lightning is presented. Lightning is caused by the build-up of electrostatic charge in clouds. As a consequence, the bottom of the cloud usually gets positively charged and the top gets negatively charged, and the ground below the cloud gets negatively charged. When the corresponding electric field exceeds the breakdown strength value of air, a disruptive discharge occurs: this is lightning.



Idealized current pulse flowing between the cloud and the ground during lightning.

Answer the following questions with the aid of this simplified curve for the current as a function of time and these data:

Distance between the bottom of the cloud and the ground: h = 1 km;
Breakdown electric field of humid air: $E_0 = 300\,\text{kV m}^{-1}$;
Total number of lightning striking Earth per year: 32×10^6;
Total human population: 6.5×10^9 people.

3.9 (0.5) What is the total charge Q released by lightning?

3.10 (0.5) What is the average current I flowing between the bottom of the cloud and the ground during lightning?

3.11 (1.0) Imagine that the energy of all storms of one year is collected and equally shared among all people. For how long could you continuously light up a 100 W light bulb for your share?

Capillary Vessels Let us regard blood as an incompressible viscous fluid with mass density μ similar to that of water and dynamic viscosity η = 4.5 g m^{-1} s^{-1}. We model blood vessels as circular straight pipes with radius r and length L and describe the blood flow by Poiseuille's law,

$$\Delta p = RD\,,$$

the Fluid Dynamics analog of Ohm's law in Electricity. Here Δp is the pressure difference between the entrance and the exit of the blood vessel, $D = S\upsilon$ is the volume flow through the cross-sectional area S of the blood vessel and υ is the blood velocity. The hydraulic resistance R is given by

$$R = \frac{8\eta L}{\pi r^4}\,.$$

For the systemic blood circulation (the one flowing from the left ventricle to the right auricle of the heart), the blood flow is $D \approx 100$ cm^{3}s^{-1} for a man at rest. Answer the following questions under the assumption that all capillary vessels are connected in parallel and that each of them has radius r = 4 µm and length L = 1 mm and operates under a pressure difference Δp = 1 kPa.

3.12 (1.0) How many capillary vessels are in the human body?

3.13 (0.5) How large is the velocity υ with which blood is flowing through a capillary vessel?

Skyscraper At the bottom of a 1000 m high skyscraper, the outside temperature is T_{bot}= 30°C. The objective is to estimate the outside temperature T_{top} at the top. Consider a thin slab of air (ideal nitrogen gas with adiabatic coefficient γ = 7/5) rising slowly to height z where the pressure is lower, and assume that this slab expands adiabatically so that its temperature drops to the temperature of the surrounding air.

3.14 (0.5) How is the fractional change in temperature dT/T related to dp/p, the fractional change in pressure?

3.15 (0.5) Express the pressure difference dp in terms of dz, the change in height.

3.16 (1.0) What is the resulting temperature at the top of the building?

Data Boltzmann constant: $k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Mass of a nitrogen molecule: $m = 4.65 \times 10^{-26}\,\mathrm{kg}$
Gravitational acceleration: $g = 9.80\,\mathrm{m\,s^{-2}}$

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Answer Script

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Digital Camera

3.1 The best spatial resolution is

(formula:) $\Delta x_{\text{min}} =$

0.7

which gives
(numerical:) $\Delta x_{\text{min}} =$

0.3

for $\lambda = 500\,\text{nm}$.

3.2 The number of Mpix is

$N =$

0.5

3.3 The best F-number value is

$F\# =$

0.5

3.4 The minimal distance is

$z =$

0.5

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Hard-boiled egg

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3.5 The required energy is

$U =$

0.5

3.6 The heat flow is

$J =$

0.5

3.7 The heat power transferred is

$P =$

0.5

3.8 The time needed to hard-boil the egg is

$\tau =$

0.5

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Lightning

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3.9 The total charge is

$Q =$

0.5

3.10 The average current is

$I =$

0.5

3.11 The light bulb would burn for the duration

$t =$

1.0

Capillary Vessels

3.12 There are

$N =$

capillary vessels in a human body.

1.0

3.13 The blood flows with velocity

$\upsilon =$

0.5

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Skyscraper

3.14 The fractional change in temperature is

$$\frac{dT}{T} =$$

3.15 The pressure difference is

$$dp =$$

3.16 The temperature at the top is

$$T_{\text{top}} =$$

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0.5

0.5

1.0