1 Oscillating rope

A) It is evident from the figure that the curvature of the rope in the fundamental vibration is very small. It infers for a possibility to model the fundamental vibration as a swinging of a rigid uniform rod of length L about a pivot point at its end. The moment of inertia of the rod is:

$$I = mL^2/3$$

and the distance from the center-of-mass to the pivot point is:

$$b = L/2$$

Therefore, the frequency of the fundamental vibration is approximated as:

$$f_1 = \frac{1}{2\pi}\sqrt{mgb/I} = \frac{1}{2\pi}\sqrt{3g/2L} \approx 0.61 \text{ Hz}$$

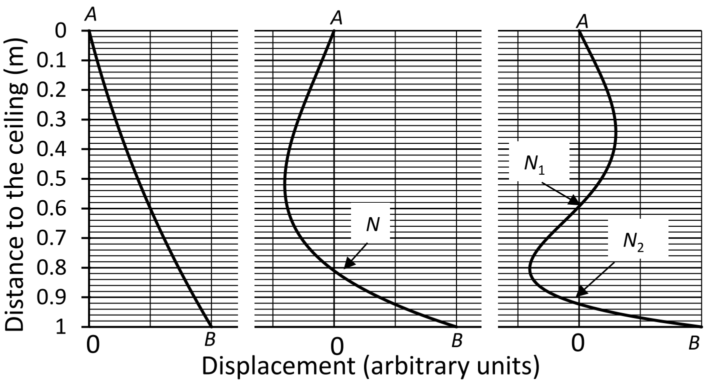
Correspondingly, the period of the fundamental vibration is:

$$T_1 = 2\pi\sqrt{I/mgb} = 2\pi\sqrt{2L/3g} \approx 1.6 \text{ s}$$

B) Whatever model for estimating of f_1 is being used, one may deduce on the basis of dimensionality arguments that the k-th natural frequency of the rope is:

$$f_k = c_k\sqrt{g/L}$$

where c_k is a dimensionless numeric coefficient depending on the consecutive mode number k only. Let A and B be the suspension point and the free end of the rope respectively, and N be the node on the rope for the second natural vibration (see the figure).



Since the node point is at rest (in the small-amplitude approximation), the vibration of the part NB could be considered as a fundamental vibration of a rope of length LNA about a suspension point N. Therefore:

$$f_2(L) \equiv f_1(L - NA)$$

Hence one may write:

$$\frac{f_2(L)}{f_1(L)} = \frac{f_1(L-NA)}{f_1(L)} = \sqrt{\frac{L}{L-NA}}$$

Since the absolute displacement is much smaller than the length of the rope, the distances could be measured in a vertical direction, to the ceiling, instead along the rope. Therefore, by taking $L = 1$ m, and $NA \approx 0.8$ m, we obtain:

$$\frac{f_2}{f_1} \approx 2.2$$

Similarly, the vibration of the part N_1B in the third eigenmode is equivalent to the second natural vibration of a rope of length $L - N_1A \approx 0.4$ m. In analogy to the first case:

$$f_3(L) \equiv f_2(L - N_1A)$$

and

$$\frac{f_2(L)}{f_1(L)} = \frac{f_2(L-N_1A)}{f_2(L)} = \sqrt{\frac{L}{L-N_1A}} \approx 1.6$$

Therefore:

$$f_3/f_1 = f_2/f_1 \times f_3/f_2 \approx 3.5$$

Finally:

$$f_1 : f_2 : f_3 \approx 1 : 2.2 : 3.5$$

Part	Marking scheme	Points
A	States explicitly or realizes (proper drawing or notattions) the physical pendulum analogy	1.0
	Correct expression for the moment of unertia	1.0
	Determination of the position of the center of mass	0.5
	Correct formula for the period/frequency of a physical pendulum	1.0
	Calculates f_1 = 0.61 Hz with two significant digits	0.5
	Subtotal on A	**4.0**
B	States or realizes (proper graph or formula) that the vibration of the rope below a node point is similar to a lower order vibration of a shorter rope.	1.0
	Uses dimensionality arguments to argue that $f_k = c_k(g/L)^{1/2}$	1.0
	Applies similarity arguments to f_1 and f_2 and derives $f_2/f_1 = (L/L-NA)^{1/2}$	1.0
	Reads correctly NA from the graph	0.5
	Calculates f_2/f_1 = 2.2 to a precision of 2 significant digits	0.5
	Applies similarity arguments to f_3 and f_2 (or f_1) and derives $f_3/f_2 = (L/L-N_1A)^{1/2}$ or $f_3/f_1 = (L/L-N_2A)^{1/2}$	1.0
	Reads correctly N_1A (N_2A) from the graph.	0.5
	Calculates f_3/f_1 = 3.5 to a precision of 2 significant digits.	0.5
	Subtotal on B	**6.0**

2 Disk in gas

The initial pressure on the thermal insulating layer is $P_0 = nk_BT_0$, where n is number density of the gas. It originates from multiplying the flux $j_0 \propto v_{x0}$ and momentum that one molecule transfers $p_0 = 2mv_{x0}$ (elastic collision), where v_{x0} is the normal component of molecule's velocity, and taking the average ($\overline{2v_{x0}^2} \propto T_0$). When applying the same idea to the surface with good thermal contact, we find out that the flux remains the same, although the momentum increases:

$$p_1 = m(v_{x0} + v_{x1}) \approx mv_{x1},$$

where v_{x1} is the normal velocity component of the molecule flying away from the disk. Thus for pressure P_1:

$$\frac{P_1}{P_0} = \frac{\overline{v_{x0}v_{x1}}}{\overline{2v_{x0}^2}} \approx \frac{\sqrt{T_1T_0}}{T_0},$$

which is correct to some numerical coefficient of the order of one.

The net force acting on the disk:

$$F = (P_1 - P_0)S \approx Snk_B\sqrt{T_0T_1},$$

and then the initial acceleration:

$$a_0 \approx \frac{Snk_B}{M}\sqrt{T_0T_1} = \frac{S\rho k_B}{mM}\sqrt{T_0T_1}.$$

Since $P_1 \gg P_0$, the disk will accelerate until its speed becomes of the order of average gas molecules speed. After the velocity v of the disc becomes on the order of $v_0 = \sqrt{kT_0/m}$, the flux of molecules reaching the backside $j(v)$ decays faster than exponentially due to the nature of the molecular velocity distribution in the ideal gas (for example, $j(2v_0) \approx 10^{-3} j_0$ and $j(3v_0) \approx 10^{-6} j_0$). That leads to a proportional decrease in a propelling pressure P_1. In order to compensate for an initial bias $\sqrt{T_1/T_0} \approx 30$, it will take around a factor of one on the velocity of the disk. Therefore the maximum velocity of the disk:

$$v_{\max} \approx v_0 = \sqrt{\frac{k_B T_0}{m}}.$$

Here we assumed that the disk will not cool close to T_0 before it reaches the maximum velocity. Let us show it. The acceleration time is approximately:

$$t_a \approx \frac{v_{\max}}{a_0} \approx \frac{M\sqrt{mk_B T_0}}{S\rho k_B \sqrt{T_0 T_1}} = \frac{M}{\rho S} \Big/ \sqrt{\frac{k_B T_1}{m}}$$

Since the power of heat removal P_{th} is maximal at the beginning (at zero velocity), we can upper-bound estimate the time for the disk to cool as $t_c = Q/P_{\text{th}}$, where Q is the total heat of the disk. The initial thermal power of heat removal can be estimated as:

$$P_{\text{th}} \approx S j_0 \times k_B T_1 \approx S n k_B \sqrt{T_0 T_1} \sqrt{\frac{k_B T_1}{m}}$$

and the total heat $Q \approx N k_B T_1$. Given $M \approx Nm$, we obtain:

$$t_c \approx \frac{(M/m) k_B T_1}{S n k_B T_1 \sqrt{k_B T_0/m}} = \frac{M}{\rho S} \Big/ \sqrt{\frac{k_B T_0}{m}}$$

Finally, $t_a/t_c \approx \sqrt{T_0/T_1} \ll 1$, and indeed disk will not cool significantly before it reaches the velocity about v_0.

Grading scheme. Indented lines show partial points for partially correct solutions

Initial acceleration (5 pts)

$P_0 = nkT = j_0 \times \Delta p_0$, $\Delta p_0 = 2mv_{x0}$ .... **2 pts**
  $P \propto kT$ .... 1 pts
$\Delta p_1 = mv_{x1}$, $P_1 \approx nk\sqrt{T_0 T_1}$ .... **2 pts**
  $P_1 = nkT_1$, no points for a_0 further .... 1 pts
  only $\Delta p_1 = mv_{x1}$ .... 1 pts
Answer for a_0 .... **1 pts**
  if some slight mistake .... 0.5 pts
Special rule
  if $\langle v_{x0} \rangle = \sqrt{3k_B T/m}$ (student doesn't understand the difference between velocity of the molecule and the component of the velocity) .... −0.5 pts

Maximal velocity (4 pts)

P_1 and P_0 depend on the velocity of the disk, P_1 drops significantly if $v \approx v_0$, thus $v_{\max} \approx v_0$ .... **4 pts**
  $P_0' \approx \rho v^2$, but P_1 stays the same, thus $v_{\max} \approx v_0\sqrt{T_1/T_0}$ .... 2 pts
  Student understands that at least some pressure depends on the velocity of the disk .... 1 pts
  The velocity is maximal when disk cools to T_0 0 pts

Justification of slow cooling (1 pts)

Estimation of times t_a and t_c given .... **1 pts**
  Student explicitly writes that $T_1' \approx T_1$ but doesn't prove it .... 0.5 pts

3 Superconducting mesh

The most important physics to consider is that the magnetic flux through the superconducting mesh is effectively locally locked in place. Consider this effect before anything else. Once the mesh is cooled to the superconducting state the magnetic field as a function of position on the mesh cannot be varied, regardless of the change in location of the dipole. Since the magnetic field is effectively specified along this superconducting plane, the problem reduces to a boundary value problem that is traditionally solved by the method of images.

First, consider what happens if the physical dipole is moved far away from the mesh. An image dipole must be located that fixes the magnetic field to be unchanged. This can be done with an image dipole that is located a distance a behind the mesh, and it must have the same orientation m. Now bring back the original dipole, placing it a distance b. It is necessary to cancel out the field from this original, but now displaced, dipole with an opposite dipole $-m$ placed behind the mesh at a distance b.

Double check your work. If the original dipole is placed at the original location a, then there is no need for image charges, and they should cancel out. Indeed, the two image dipoles will, as they have opposite orientations.

The force between the dipole and the image charges must be determined. Though it might be possible to write down these answers quickly, the derivation is shown below.

Consider first that a magnetic dipole moment m can be thought of as a pair of magnetic monopoles of strength q_m and $-q_m$ separated by a distance d such that $m = q_m d$. Determine the magnetic field strength a distance $x \gg d$ away from the dipole:

$$B = \frac{\mu_0}{4\pi}\frac{q_m}{x^2} + \frac{\mu_0}{4\pi}\frac{-q_m}{(x+d)^2}.$$

It should be clear that q_m is at the origin and $-q_m$ is a distance d farther away from the reference point x where the field B is being determined. This expression is exact.

The second term can be subjected to a binomial expansion and then

$$B \approx \frac{\mu_0}{4\pi}\frac{q_m}{x^2} - \frac{\mu_0}{4\pi}\frac{q_m}{x^2}\left(1 - 2\frac{d}{x}\right) = \frac{\mu_0}{2\pi}\frac{q_m d}{x^3} = \frac{\mu_0}{2\pi}\frac{m}{x^3}$$

Now consider the magnetic force on a dipole at the location x in a non-uniform field B, which is given by

$$F = -q_m B(x) + q_m B(x+d)$$

which can be approximated by a Taylor expansion of B,

$$F \approx -q_m B(x) + q_m \left(B(x) + d \left.\frac{dB}{dx}\right|_x \right)$$

$$= q_m d \left(-\frac{3\mu_0}{2\pi}\frac{m}{x^4} \right) = -\frac{3\mu_0}{2\pi}\frac{m^2}{x^4}$$

The negative sign means that two parallel identical dipoles separated by a distance x will attract.

Returning to the problem, the physical dipole at b will be attracted to the image dipole at location $-a$ and repelled from the image dipole at $-b$, so

$$F = -\frac{3\mu_0}{2\pi}\frac{m^2}{(b+a)^4} + \frac{3\mu_0}{2\pi}\frac{m^2}{(b+b)^4} = \frac{3\mu_0 m^2}{2\pi}\left(\frac{1}{16b^4} - \frac{1}{(a+b)^4}\right),$$

where a negative sign means that the physical dipole feels attraction toward the mesh.

It is entertaining to consider what happens if b is almost the same as a, say $b = a + \delta$. In this case,

$$F = \frac{3\mu_0 m^2}{2\pi}\left(\frac{1}{16(a+\delta)^4} - \frac{1}{(2a+\delta)^4}\right),$$

or

$$F \approx \frac{3\mu_0 m^2}{2\pi}\frac{1}{16a^4}\left(\left(1 - 4\frac{\delta}{a}\right) - \left(1 - 4\frac{\delta/2}{a}\right)\right),$$

which simplifies further into

$$F \approx -\frac{3\mu_0 m^2}{16\pi a^5}\delta.$$

Now to interpret. A negative force here is a force of attraction toward the mesh. A positive δ is moving the physics dipole away from the mesh. As such, the force is a linear restoring force, and slight disturbances to the physical dipole will result in simple harmonic oscillations about the original position.

Grading scheme

1.5 Recognition of nature of problem

- Recognize flux trapping in superconductor (1.0 pt)
- Recognize that flux trapping creates a boundary value problem (0.5 pt)

4.0 Recognize that the boundary value problem requires two image dipoles

- First image dipole to create original B field on mesh (0.5 pt)
- Correct location of first image dipole (0.5 pt)
- Correct magnitude of first image dipole (0.5 pt)
- Correct orientation of first image dipole (0.5 pt)
- Second image dipole to cancel new B field on mesh (0.5 pt)
- Correct location of second image dipole (0.5 pt)
- Correct magnitude of first image dipole (0.5 pt)
- Correct orientation of first image dipole (0.5 pt)

2.0 Determine the force between two dipoles

- Determine B field a distance from a dipole (1 pt)
- Determine force on a dipole in a non-uniform B field (1 pt)

2.5 Determine the force between the physical dipole and the mesh

- Correct magnitude and direction of force from image dipole one (1 pt)
- Correct magnitude and direction of force from image dipole two (1 pt)
- Correct magnitude and direction of force (0.5 pt)

Some notes:

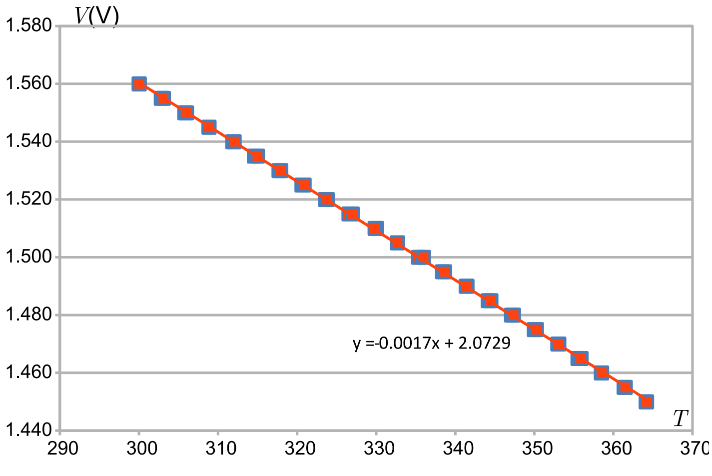
- Dimensionally correct expression with no shown work but have wrong prefactor get zero marks
- Dimensionally correct expression that show work but have wrong prefactor caused from clear trivial math mistake get 1/2 marks.
- Dimensionally correct expression that whow work but have wrong prefactor caused from serious math mistake or any physics mistake get zero marks.
- Dimensionally incorrect expression with no shown work get zero marks
- Dimensionally incorrect answers that show work get zero marks
- Follow on errors that use dimensionally correct, but wrong, derived inputs are not penalized further.
- Follow on errors that use dimensionally incorrect derived inputs are penalized half marks each time the input is used.
- Writing a formula incorrectly without showing the derivation and using it as an input is not considered a follow on error, but is instead a non-trivial error with a result of zero marks for that part.
- Ambiguous locations, magnitudes, or orientations receive zero marks.
- Correctly identifying the locations, magnitudes, or orientations of both of the two image dipoles without clearly specifying why will receive full 1.5 points for recognizing the nature of the problem.
- Correctly identifying the locations, magnitudes, or orientations of only one of the image dipoles without clearly specifying why will receive 0.5 points for recognizing the nature of the problem.

1 Light-emitting diode

Task 1. The easiest way to obtain $V(T)$-graph is to use the diode function of the multimeter for measuring diode voltage V. The board is heated by driving through the resistor R_1 a current which is adjusted using the potentiometer.

V (V)	R_T (kΩ)	T (K)	V (V)	R_T (kΩ)	T (K)
1.560	9.25	300.0	1.560	9.25	300.0
1.555	8.18	302.8	1.555	8.08	303.1
1.550	7.2	305.8	1.550	7.13	306.0
1.545	6.33	308.9	1.545	6.33	308.9
1.540	5.6	311.9	1.540	5.56	312.0
1.535	5.02	314.6	1.535	4.94	315.0
1.530	4.44	317.7	1.530	4.4	317.9
1.525	3.97	320.6	1.525	3.93	320.9
1.520	3.55	323.6	1.520	3.52	323.8
1.515	3.18	326.6	1.515	3.13	327.0
1.510	2.8	330.1	1.510	2.82	329.9
1.505	2.55	332.7	1.505	2.55	332.7
1.500	2.32	335.4	1.500	2.27	336.0
1.495	2.09	338.4	1.495	2.07	338.7
1.490	1.88	341.5	1.490	1.888	341.4
1.485	1.72	344.1	1.485	1.7	344.5
1.480	1.56	347.1	1.480	1.546	347.4
1.475	1.42	350.0	1.475	1.407	350.3
1.470	1.29	353.0	1.470	1.285	353.1
1.465	1.19	355.5	1.465	1.175	355.9
1.460	1.085	358.5	1.460	1.083	358.5
1.455	0.992	361.4	1.455	0.987	361.5
1.450	0.911	364.1	1.450	0.909	364.2

Once we estimate the magnitude of the terms in the expression for I_d, we'll find that $V_T \approx 25\,\text{mV}$ for $T \approx 300\,\text{K}$; all the voltages are much larger than that, so the unity can be neglected. Then, with constant I_d, we have $\frac{V-V_{G0}}{nV_T} = \text{const}$ (the constant appears to be negative), hence $V = V_{G0} - BnV_T = BnkT/q$, where B is a constant. So, we need to plot V versus T, and V_{G0} is found as the intersection point of the linear regression line with the vertical axis.



From the linear regression we obtain $v_{G0} \approx 2.085\,\text{V}$.

Next we can make a series of measurements with small currents so that the diode will have essentially the room temperature. Then we can take logarithm from the expression of I_d (while neglecting the unity) to obtain

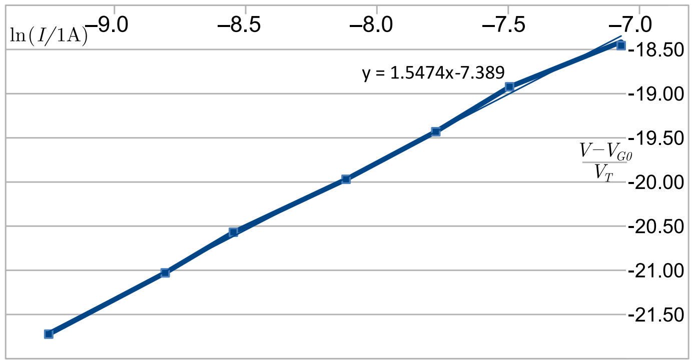
$$\frac{V - V_{G0}}{V_T} = n \ln A - n \ln I_d.$$

Thus, if we plot $\ln I_d$ versus $V - V_{G0}$, n and A can be obtained as the linear regression parameters.

Alternatively, we can make a series of measurements with a fixed voltage applied to the diode, and measure current for different temperatures. Based on the same expression as given above, we need to plot $\frac{V-V_{G0}}{V_T} = \frac{(V-V_{G0})q}{kT}$ versus $\ln I_d$. Such data are given in the table below.

I (mA)	R_T (kΩ)	T (K)	$\ln(I/1\,\text{A})$	$\frac{(V-V_{G0})q}{kT}$
33.3	9.76	298.8	-10.310	-22.25
44.8	5.51	312.3	-10.013	-21.29
96.6	7.12	306.1	-9.245	-21.72
150.0	4.72	316.1	-8.805	-21.03
194.1	3.6	323.2	-8.547	-20.57
297.0	2.53	332.9	-8.122	-19.97
417.0	1.84	342.1	-7.782	-19.43
551.0	1.36	351.3	-7.504	-18.92
840.0	1.002	361.0	-7.082	-18.41
860.0	0.95	362.8	-7.059	-18.33

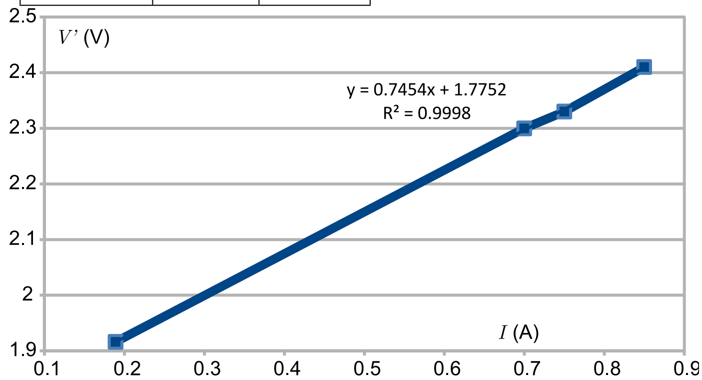
If we plot these data we'll see that most of the data points lie on a straight line, but first two and the very last one will deviate. Data points corresponding to very small currents deviate because of large relative uncertainties, last data point deviates because the parasitic resistance can no longer be neglected. So we discard these from our analysis.



With this graph, the tangent of the slope of the regression line gives us directly $n \approx 1.55$. The intercept -7.39 gives us $-n \ln A$, so that $A \approx e^{7.39/1.55}\text{A} \approx 119\text{A}$.

By very large currents, a certain change in current ΔI_d will give rise to just a tiny change in the voltage drop ΔV at the diode junction, if we can keep the temperature constant by compensating with the resistor. [according to the exponential dependence of $I_d = I_d(V)$], which is much smaller than the change in the voltage drop on the parasitic resistor $R_s \Delta I_d$. So, we can determine R_s from the experimentally measured dependance of the diode current I_d on the total voltage $V' = V + I_d R_s$ at the limit of large currents.

R_T (kΩ)	I_d (A)	V' (V)
0.986	0.85	2.41
1.435	0.7	2.3
1.360	0.75	2.33
6.55	0.189	1.916



The regression line slope gives directly $R_s \approx 0.75\,\Omega$.

Other option is to measure the voltage, temperature and current (at high current values) and substract the diode voltage calculated from the model to get the voltage drop on series resistance.

In both cases, it is important that the voltage is measured directly from the LED wires, to ignore the any additional voltage drops on high current carrying part of circuit.

Task 2. The idea is to compare the thermal expansion of air inside the bottle when the heat is being released by the diode, and when this is done by the resistor. In the latter case, all the consumed electrical power is released as heat; in the former case, part of the heat escapes the bottle as light radiation energy. The pressure inside a bottle is a function of its temperature, and the temperature which will establish inside the bottle is defined by the balance between the thermal dissipation power, and the rate by which heat is escaping the bottle. The latter is a function of the temperature inside the bottle, and hence, the temperature inside the bottle is a function of the heat dissipation power. Therefore, the pressure inside the bottle is also a function of the heat dissipation power.

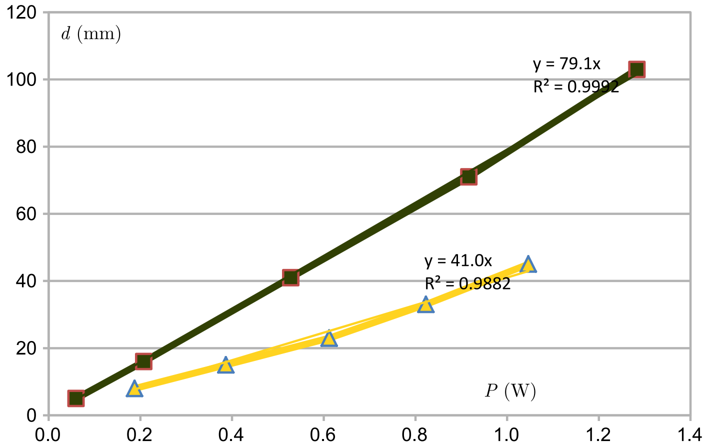
First we need to build a manometer which allows us to measure pressure difference — either between the pressure inside the bottle and the atmospheric pressure, or between the pressure inside the two bottles. To that end, we fill a tube with water and folding it into a U-tube; pressure difference can be determined via the water column height difference between two halves of the tube.

One possible approach to this problem is to compare heat dissipation on the diode with a heat dissipation on the resistor: one may adjust (using the potentiometer) the heating power of the resistor to reach such a state that the pressures inside the bottles are equal. If this state is reached when the electrical consumption power of the diode is P_D and the power of the resistor is P_R then we can conlcude that thermal dissipation power of the diode is also P_R, and hence, $P_D - P_R$ escapes the bottle as the light radiation so that $\eta = 1 - P_R/P_D$.

Thermal equilibrium inside the bottles is reached relatively slowly, one must wait aproximately for 5-10 minutes. Therefore, finding the state when the heat dissipation powers on the diode and on the resistor are equal may be a relatively slow process. An alternative approach is to assume that the heat exchange rate with the surrounding to be linear in temperature (this assumption is well-founded as the diode never becomes very hot). Then we can make two series of measurements for the pressure inside the bottle as a function of electrical consumption power: first, when the current is driven through the diode, and second, when it is driven through the resistor, $\Delta p = \Delta p(P)$. Fitting the data to a linear law, $\Delta p = kP$, we obtain the values of the proportionality coefficient k both for the diode (k_D) and for the resistor (k_R); then, $\eta = 1 - k_R/k_D$. Note that since the pressure is proportional to the column height difference, we can express the pressure in terms of the height of water column: the proportionality factor cancels out from the ratio.

Measurement data are given in the table below; I, V, P, and d refer to the measured current, voltage, power, and water column height difference, respectively; index "L" refers to the measurements with a diode, and "R" — to the measurements with a resistor.

I_D	V_D	P_D	d_D	I_R	U_R	P_R	d_R
101.5	1.85	0.1878	8	101.9	0.585	0.0596	5
198.2	1.95	0.3865	15	190.1	1.09	0.2072	16
300	2.04	0.6120	23	300	1.76	0.5280	41
390	2.11	0.8229	33	390	2.35	0.9165	71
480	2.18	1.0464	45	470	2.73	1.2831	103

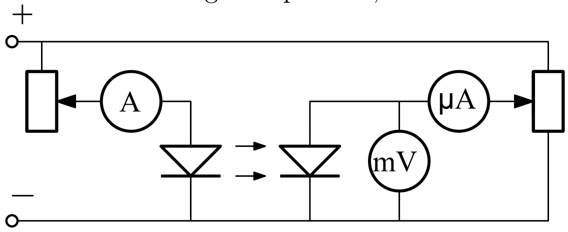


Based on these slopes $k_D = 1 - 41.0\,\text{mm/W}/79.8\,\text{mm/W} \approx 0.48$

Note that it is also possible to measure the efficiency using the temperature sensor: it works in the same way as the method using air expansion described above. We compare temperature of the circuit board as function of electrical power supplied to the diode [$T_D(P)$], and also the temperature when power is supplied to the resistor [$T_R(P)$]. Assuming that dominating part of the heat is dissipated into surroundings via the circuit board and only a negligible part of it leaves as a heat radiation at the diode and resistor, respectively (this is an assumption which is valid with a really good accuracy), we can find $\eta = 1 - T_D(P)/T_R(P)$, or even better, $\eta = 1 - \kappa_D(P)/\kappa_R(P)$, where κ_D and κ_R denote the slopes of the respective graphs. The result is the same as shown above, $\eta \approx 0.46 \pm 0.04$. It is also possible to calculate the internal efficiency of the junction by subtracting the power dissipation on the parasitic resistance; the result is $\eta_{\text{internal}} \approx 0.53$.

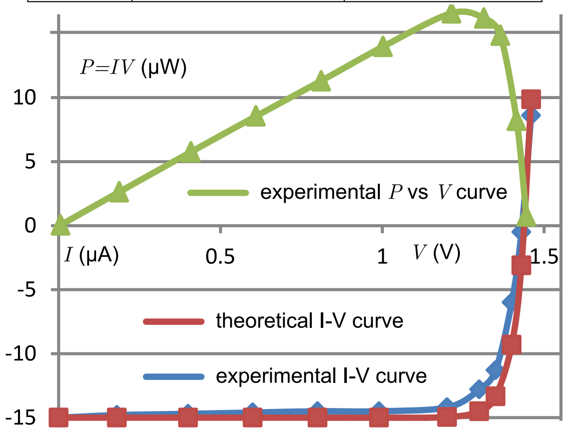
Task 3. According to our model, the photocurrent does not depend on the voltage, but because the diode current does, the total current through the diode depends on the voltage. For the maximum harvestable electrical power P_{max} we must find a voltage where $P = VI_p - VI_d$ is the greatest.

We can do it in multiple ways. One way is to measure the $I_p \approx 0.020\,\text{mA}$ by shorting the diode with a am-meter, and then finding the maximum analytically or numerically from our model. Using the diode parameters from above we get $P_{\max} \approx 0.026\,\text{mW}$. Other way is to change the voltage on the diode with potentiometer, measure the current and the voltage and find the maximum by scanning the range where current and voltage are positive, see the circuit below.



The data of measurements are given in the table below; the graph shows both the experimental data points and theoretical dependence $I_d(V) - I_p$, where $I_p = -15\,\mu\text{V}$, determined using the measurement at $V = 0\,\text{V}$. As we can see, the two curves are fairly close; there is a small mismatch which can partly explained by a leakage current due to a certain large effective resistance being connected in parallel to the diode's junction.

V (V)	$I = I_d - I_p$ (A)	$P = VI$ (μW)
0	-15	0
0.18	-14.8	2.664
0.4	-14.7	5.88
0.6	-14.6	8.76
0.8	-14.5	11.6
0.99	-14.5	14.355
1.2	-14.2	17.04
1.3	-12.8	16.64
1.35	-11.3	15.255
1.4	-6	8.4
1.43	-0.5	0.715
1.46	8.6	-12.556



The graph includes also the curve for the electrical power produced by the diode, $P = VI$. The maximum $P_{\max} \approx 17\,\mu$W can be determined as the maximum of this curve. The efficiency is found as

$$\eta_p = \frac{P_{\max}}{P_i} = \frac{P_{\max}}{\eta I_1 V(I_1)\frac{S}{\alpha 4\pi d^2}},$$

where $\frac{S}{\alpha 4\pi d^2}$ is fraction of the light reaching the active area of the LED; V_1 and I_1 denote the voltage and current of the other diode, respectively. Numerically we obtain $\eta_p \approx 0.04$.

Coarse-grained grading scheme

1 Voltage-temperature graph		
Plot	Method	0.5
	Measurements	1.0
	Plot	0.5
VG0	Method	1.0
	Value	0.5
n,A	Method	1.0
	Measurements and plot	1.5
	Value for n	0.5
	Value for A	0.5
Rs	Method	1.0
	Measurements	0.5
	Value	0.5
2 Efficiency		
η	Method	2.0
	Measurements	2.0
	Value	1.0
3 LED as photodiode		
Nmax	Method	2.0
	Measurements	2.0
	Value	0.5
ηp	Formula	1.0
	Value	0.5