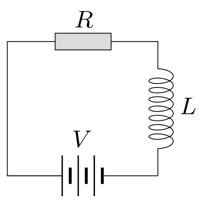
T1: Floating cylinder (10 pts)

A solid, uniform cylinder of height $h = 10$ cm and base area $s = 100$ cm^2 floats in a cylindrical beaker of height $H = 20$ cm and inner bottom area $S = 102$ cm^2 filled with a liquid. The ratio between the density of the cylinder and that of the liquid is $\gamma = 0.70$. The bottom of the cylinder is above the bottom of the beaker by a few centimeters. The cylinder is oscillating vertically, so that its axis always coincides with that of the beaker. The amplitude of the liquid level oscillations is $A = 1$ mm.

Find the period of the motion T. Neglect the viscosity of the liquid.

T2: Thermal oscillations (10 pts)

A resistor is made of a material which undergoes a phase transition so that its resistance takes one of the two values, R_1 if its temperature is smaller than T_c, and $R_2 > R_1$ if the temperature is larger than T_c.



This resistor is connected to a voltage source through an inductor of inductance L. It appears that if the applied voltage V is between two critical values, $V_1 < V < V_2$, the temperature of the resistor starts oscillating. Assume that (i) the heat flux P from the resistor to the ambient medium is given by $P = \alpha(T - T_0)$, where α is a constant, T denotes the temperature of the resistor, and T_0 is the ambient temperature; (ii) the geometrical size of the resistor is so small that it will reach a thermal equilibrium much faster than the characteristic time L/R_2.

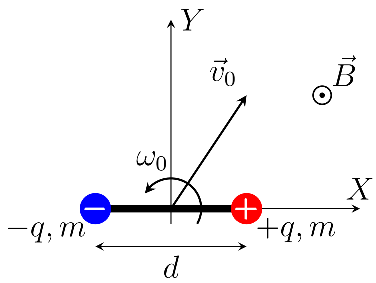
(a) *(2 pts)* Express V_1 and V_2 in terms of the other parameters defined above.

(b) *(6 pts)* Assuming that $V_1 < V < V_2$, sketch qualitatively how the temperature of the resistor T depends on time t, and find the ratio $(T_{\text{max}} - T_0)/(T_{\text{min}} - T_0)$, where T_{max} and T_{min} denote the maximal and minimal values of T, respectively.

(c) *(2 pts)* Find the period of oscillations if $V = \sqrt{V_1 V_2}$ and $R_2 = 16R_1$.

T3: Dipole in a magnetic field (10 pts)

Two small balls of mass m each with charges $+q$ and $-q$ respectively, connected by a rigid massless rod of length d, form a dipole. The dipole is parallel to plane XY and is placed in a uniform magnetic field $\vec{B}$ perpendicular to XY.



Initially, the dipole is aligned with the direction X and has initial angular velocity ω_0 in plane XY, as shown. Its center of mass is initially located at origin and given initial velocity $\vec{v}_0$ parallel to XY, as well.

Consider three distinct scenarios (a, b, c-d):

(a) *(2 pts)* Find ω_0 and the direction of $\vec{v}_0$, so that the center of mass will move with the constant velocity $\vec{v} = \vec{v}_0$?

(b) *(3 pts)* Given ω_0, find such $\vec{v}_0$ (direction and magnitude), so that the center of mass will travel in a circle. Find the circle radius R_c and the coordinates x_c, y_c of its center. You don't need to prove the uniqueness of the solution.

(c) *(4 pts)* Given $\vec{v}_0 = 0$, find the minimal $\omega_0 = \omega_{\text{min}}$ necessary for the dipole to reverse its orientation during the motion.

(d) *(1 pt)* If the dipole starts with $\vec{v}_0 = 0$ and $\omega_0 = \omega_{\text{min}}$ found in part (c), the trajectory of its center of mass has an asymptote. Find the distance D from the origin to the asymptote.

Useful vector identity:

$$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}\,(\vec{a} \cdot \vec{c}) - \vec{c}\,(\vec{a} \cdot \vec{b}),$$

where "$\times$" and "$\cdot$" denote vector product and scalar product respectively.