Marking Scheme Q1 (10 points)

Part A (3.0 pt) If the final answer is written then the complete point will be achieved

A-1	$S_0 = \sigma T_S^4 . \left(\frac{R_S}{d}\right)^2$ (0.4pt), [Realizing energy conservation (0.1pt)] Numerical value of $S_0 = 1.35 \times 10^3\ \mathrm{W/m^2}$ (0.2pt) [more than 4 significant figures (0.1pt)]	0.6 pt
A-2	$T_E = \left(\frac{S_0}{4\sigma}\right)^{\frac{1}{4}} = \sqrt{\frac{R_S}{2d}} T_S$ (0.4pt), [realizing energy balance (0.1pt)] Numerical value of $T_E = 278\ \ \mathrm{K}$ (0.2pt) [more than 4 significant figures (0.1pt)]	0.6 pt
A-3	$f(x) = 5(1 - e^{-x}) - x$	0.4 pt
A-4	$x_m = \{4.96, 4.97\}$ (0.3 pt), [more than 4 significant figures (0.2pt)] Numerical value of $b = [2.89, 2.90] \times 10^6\ \mathrm{nm.K}$ (0.1 pt) [more than 4 significant figures (0.1pt)]	0.4 pt
A-5	$\lambda_{max}^{Sun} = [5.01,\ 5.02] \times 10^2\ \mathrm{nm}$(0.1 pt), $\lambda_{max}^{Earth} = 1.04 \times 10^4\ \mathrm{nm}$(0.1 pt) [more than 4 significant figures (0.1pt)]	0.2 pt
A-6	$\gamma = \left(\frac{d}{R_S}\right)^2 \times \left(\frac{T_E}{T_S}\right)^5 = \left(\frac{\lambda_S}{\lambda_E}\right)^5 \times \left(\frac{d}{R_S}\right)^2$ (0.6 pt), [realizing $\tilde{u}_S = \left(\frac{R_S}{d}\right)^2 u_S(\lambda)$(0.3pt)] Numerical value of $\gamma = [1.20, 1.21] \times 10^{-2}$ (0.2 pt) [more than 4 significant figures (0.1pt)]	0.8 pt

Part B (7.0 pt)

B-1	$T_A = \left(\frac{(1-r_A)\frac{S_0}{4}}{\sigma}\right)^{\frac{1}{4}}$ $T_E = \left(\frac{(1-r_A)\frac{S_0}{2}}{\sigma}\right)^{\frac{1}{4}}$ Two correct expressions (0.8 pt) [One correct expression (0.6 pt)] [no correct expression: for each energy balance relation (0.2pt)] Numerical value of $T_A = 2.58 \times 10^2\ \mathrm{K}$ (0.1 pt) Numerical value of $T_E = 3.07 \times 10^2\ \mathrm{K}$ (0.1 pt) [more than 4 significant figures (0.1pt)]	1.0 pt
B-2	$\alpha = r_A + \frac{(1-r_A)^2 r_E}{1-r_A r_E}$ (1.4pt) [$\tilde{S}_0$=$r_A S_0$ (0.1 pt) $\tilde{S}_1 = (1-r_A)^2 r_E S_0 = \frac{(1-r_A)^2}{r_A} r_E \tilde{S}_0$ (0.3 pt)	1.6 pt

	$\tilde{S}_n = \frac{\tilde{S}_{n-1}}{1-r_A} r_\mathrm{A} r_\mathrm{E} \times (1-r_\mathrm{A}) = r_\mathrm{A} r_\mathrm{E} \tilde{S}_{n-1} = (r_\mathrm{A} r_\mathrm{E})^{n-1} \tilde{S}_1$ (0.5 pt) $\tilde{S} = \sum_{n=0}^{\infty} \tilde{S}_n = \tilde{S}_0 + \tilde{S}_1 \sum_{n=1}^{\infty} (r_\mathrm{A} r_\mathrm{E})^{n-1}$ (0.3 pt)] Numerical value of $\alpha = 3.13 \times 10^{-1}$ (0.2pt) [more than 4 significant figures (0.1pt)]	
B-3	$T_\mathrm{E} = \left[\frac{(1-\alpha)}{2\sigma(2-\epsilon)} S_0\right]^{\frac{1}{4}}$ (0.6pt) Numerical value of $\epsilon = [8.07, 8.11] \times 10^{-1}$ (0.4pt) [wrong numerical value: correct expression for ϵ (0.2pt)] [more than 4 significant figures (0.3pt)]	1.0 pt
B-4	$\frac{dT_\mathrm{E}}{d\epsilon} = \frac{1}{4}\left[\frac{(1-\alpha)S_0}{2\sigma(2-\epsilon)}\right]^{\frac{1}{4}} \frac{1}{(2-\epsilon)}$ (0.6 pt), Numerical value of $\delta T_\mathrm{E} = [4.87, 4.92] \times 10^{-1}$ K (0.2pt) [more than 4 significant figures (0.1pt)]	0.8 pt
B-5	$\epsilon = \frac{\sigma T_\mathrm{E}^4 - (1-\alpha)\frac{S_0}{4}}{\sigma(T_\mathrm{E}^4 - T_\mathrm{A}^4)}$ (0.6pt) $k = \frac{(2T_\mathrm{A}^4 - T_\mathrm{E}^4) \times \left[\sigma T_\mathrm{E}^4 - (1-\alpha)\frac{S_0}{4}\right]}{(T_\mathrm{E}^4 - T_\mathrm{A}^4) \times (T_\mathrm{E} - T_\mathrm{A})}$ (0.6pt) [Correct relations for balance of energy (0.3+0.3 pt)] Numerical value of $\epsilon = [8.47, 8.52] \times 10^{-1}$ (0.2pt) Numerical value of $k = [3.57, 3.66] \times 10^{-1}$ W/m^{2}K (0.2pt) [more than 4 significant figures for each one (0.1pt)]	1.6 pt
B-6	(a) (0.4+0.4) $\begin{cases} \epsilon\left[\frac{1}{T_\mathrm{E}-T_\mathrm{A}} + \frac{4T_\mathrm{E}^3}{2T_\mathrm{A}^4 - T_\mathrm{E}^4}\right]\frac{dT_\mathrm{E}}{d\epsilon} = 1 + \epsilon\left[\frac{8T_\mathrm{A}^3}{2T_\mathrm{A}^4 - T_\mathrm{E}^4} + \frac{1}{T_\mathrm{E}-T_\mathrm{A}}\right]\frac{dT_\mathrm{A}}{d\epsilon} \\ 1 + \epsilon\left[\frac{4T_\mathrm{E}^3}{T_\mathrm{E}^4 - T_\mathrm{A}^4} - \frac{4\sigma T_\mathrm{E}^3}{\sigma T_\mathrm{E}^4 - (1-\alpha)\frac{S_0}{4}}\right]\frac{dT_\mathrm{E}}{d\epsilon} = \frac{4T_\mathrm{A}^3}{T_\mathrm{E}^4 - T_\mathrm{A}^4}\epsilon\frac{dT_\mathrm{A}}{d\epsilon} \end{cases}$ (0.6 pt) (b) $\delta T_\mathrm{E} = [5.21, 5.28] \times 10^{-1}$K (0.2pt) [more than 4 significant figures for each one (0.1pt)]	1.0 pt

Marking Scheme Q2 (10 points)

Part A (5.6 pt)

A-1		1.5 pt
(a)	$\vec{E}(x,y,z) = \frac{-\lambda x}{4\epsilon_0 R^2}\hat{x} + \frac{-\lambda y}{4\epsilon_0 R^2}\hat{y} + \frac{\lambda z}{2\epsilon_0 R^2}\hat{z}$ (1.0 pt) [z-component (0.5 pt), x- and y- components (0.5 pt), wrong coefficient for each component (-0.1 pt), wrong sign for each component (-0.2 pt)]	
(b)	$\omega_x = \omega_y = \sqrt{\frac{Q\lambda}{4\epsilon_0 R^2 m}}$ (0.5 pt)	
A-2	$a = \frac{Qu}{2\epsilon_0 R^2 m \Omega^2}$ (0.2 pt) $k = \sqrt{\frac{Q\lambda_0}{2\epsilon_0 R^2 m}}$ (0.2 pt)	0.4 pt
A-3	$\ddot{q} = pa\Omega^2 \cos\Omega t$ (1.0 pt) [each of the 3 approximations (0.3 pt), the final equation (0.1pt)] $q = -pa\cos\Omega t$ (0.8 pt) [general solution (0.4 pt), fixing the free parameters in the general solution each (0.2 pt)]	1.8 pt
A-4	$\ddot{p}(t) = \left(k^2 - \frac{a^2\Omega^2}{2}\right)p$ (1.2 pt) [Correct approach (0.6 pt), Correct result (0.6 pt)] $\Omega > \sqrt{2}\frac{k}{a}$ (0.3 pt)	1.5 pt
A-5	$k = 2 \times 10^5$ rad/s (0.2 pt) $\Omega_{\text{min}} \simeq 7 \times 10^6$ rad/s (0.2 pt) [inappropriate number of significant figures (-0.1 pt)]	0.4 pt

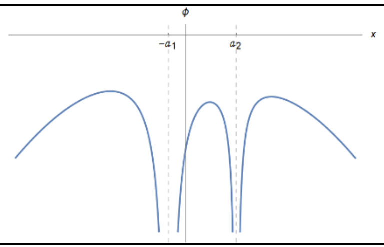
Part B (4.4 pt)

B-1	$\Gamma = \frac{1}{\tau}$ (0.5 pt) [Answers with different numerical coefficients should be considered as correct answers]	0.5 pt

B-2	$s_+ = s_L + \alpha\omega_L \frac{v}{c}$ (0.5 pt) $s_- = s_L - \alpha\omega_L \frac{v}{c}$ (0.5 pt) [correct Doppler shift each (0.3 pt), final answer each (0.2 pt)] $\pi_+ = s_+ \times (-\hbar k_+)$ (0.1 pt) $\pi_- = s_- \times (+\hbar k_-)$ (0.1 pt) $F = -(2\alpha\hbar k_L^2)v$ (0.5 pt)	1.7 pt
B-3	$\begin{cases} p = 0 \\ p = +2\hbar k_L \end{cases}$ (0.5 pt) [one correct answer (0.3 pt)] $P_{in} = \frac{\hbar^2 k_L^2}{m\tau}$ (0.5 pt)	1.0 pt
B-4	$P_{out} = -2\alpha\hbar k_L^2 v^2$ (0.3 pt) $\overline{v^2} = \frac{\hbar\Gamma}{2\alpha m}$ (0.3 pt) $T = \frac{\hbar\Gamma}{2\alpha k_B}$ (0.2 pt) [Answers with different numerical coefficients should be considered as correct answers]	0.8 pt
B-5	$T = 2 \times 10^{-4}$ K (0.4 pt) [according to the coefficient used in the part B.4, the resulting temperature might be different.]	0.4 pt

Marking Scheme Q3 (10 points)

Part A (5.0 pt)

A-1	$\Phi(x,y) = -\frac{GM_1}{\sqrt{\left(x+\frac{M_2}{(M_1+M_2)}a\right)^2+y^2}} - \frac{GM_2}{\sqrt{\left(x-\frac{M_1}{(M_1+M_2)}a\right)^2+y^2}} - \frac{1}{2}\frac{G(M_1+M_2)}{a^3}(x^2+y^2)$ [Gravitational part (0.5 pt)] [Centrifugal part (0.5 pt)]	1.0 pt
A-2	[Correct behavior at infinity (0.1 pt)] [Three maximums (0.3 pt)] [Two vertical asymptotes (0.3 pt)] 	0.7 pt
A-3	$\frac{x_0}{a} = 0.36$ [In case of obtaining correct equation but not solving it (0.2 pt)] [Obtaining the numerical result with one decimal figure (0.3 pt)]	0.5 pt
A-4	$\dot{a} = -2\beta a\left(\frac{1}{M_1} - \frac{1}{M_2}\right)$ (0.3 pt) $\dot{P} = -6\pi\sqrt{\frac{a^3}{GM}}\beta\left(\frac{1}{M_1} - \frac{1}{M_2}\right)$ (0.3 pt) [Only correct approach (conservation of momentum) (0.2 pt)]	0.6 pt
A-5	$T = \left(\frac{GM_1\beta}{8\pi\sigma r^3}\right)^{\frac{1}{4}}$ [Correct approach (Energy relation) (0.5 pt)] [Correct solution (0.5 pt)]	1.0 pt
A-6	$a = \left[\frac{P^2G(M_S+M_{NS})}{4\pi^2}\right]^{\frac{1}{3}}$ (0.3 pt) $T = \left(\frac{500\pi M_{NS}\beta}{\sigma P^2(M_S+M_{NS})}\right)^{\frac{1}{4}}$ (0.1 pt) $T = 9\times 10^3\ \mathrm{K}$ (0.1 pt) [If the final answer for T is correct the complete pt will be given]	0.5 pt
A-7	$E' = \frac{1}{2}\mu' v'^2 - \frac{GM_1'M_2}{a} < 0$ (0.2 pt) $v'_{max} = \sqrt{\frac{2G(M_1'+M_2)}{a}}$ (0.2 pt) $v' = v$ (0.2 pt) $M'_{1min} = \frac{M_1-M_2}{2}$ (0.1 pt)	0.7 pt

Part B (5.0 pt)

B-1	$g=-\frac{4\pi G\rho_c r}{3}$	0.2 pt
B-2	$h_1(\rho,r)=r^2\rho^{\gamma-2}$ $h_2(r)=\frac{4\pi G r^2}{K\gamma}$ [$\vec{F}=-\frac{GM(\vec{r})\rho}{r^2}\mathrm{A}\,\Delta r-\Delta pA=0$ (0.3 pt)]	0.6 pt
B-3	$r_0=G^{-\frac{1}{2}}p_c^{\frac{1}{2}}\rho_c^{-1}$	0.4 pt
B-4	$A_1(u,x)=x^2u^{\gamma-2}$ $A_2(x)=\frac{4\pi x^2}{\gamma}$ The answer would be correct up to a constant coefficient	0.3 pt
B-5	$f(x)=A\sin(\sqrt{2\pi}x)+B\cos(\sqrt{2\pi}x)$ (0.3 pt) $A=\frac{1}{\sqrt{2\pi}}$ (0.2 pt) & $B=0$ (0.1 pt)	0.6 pt
B-6	$u'(0)=0$ (0.1 pt) $\lim_{x\to 0}\frac{u'(x)}{x}=u''(0)$ (0.4 pt) $\gamma=-\frac{4\pi}{3u''(0)}$ (0.2 pt) $\gamma\sim 1.66$ (0.1 pt)	0.8 pt
B-7	$\tilde{\rho}\simeq\rho(1-3\epsilon)$ (0.6 pt) [$\tilde{\rho}=\rho(1+\epsilon)^{-3}$(0.4pt)] $\tilde{g}\simeq g(1-2\epsilon)$ (0.3 pt) [$\tilde{g}=g(1+\epsilon)^{-2}$(0.2pt)]	0.9 pt
B-8	$\ddot{\tilde{r}}=\tilde{g}-k\gamma\tilde{\rho}^{\gamma-2}\frac{\partial\tilde{\rho}}{\partial\tilde{r}}$	0.6 pt
B-9	$\ddot{\epsilon}=-\frac{4\pi G\rho_c}{3}(3\gamma-4)\epsilon$ (0.4 pt) $\gamma_{\min}=\frac{4}{3}$ (0.1 pt) $\omega=\sqrt{\frac{4\pi G\rho_c}{3}(3\gamma-4)}$ (0.1 pt)	0.6 pt

Problem E1- Solution

Heat Conduction in a Copper Rod (10 points)

0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9

Part A: The short copper rod (3.9 points)

A.1 (0.2 pt)

$R_{en} = 110.11 \pm 0.01\ \Omega$

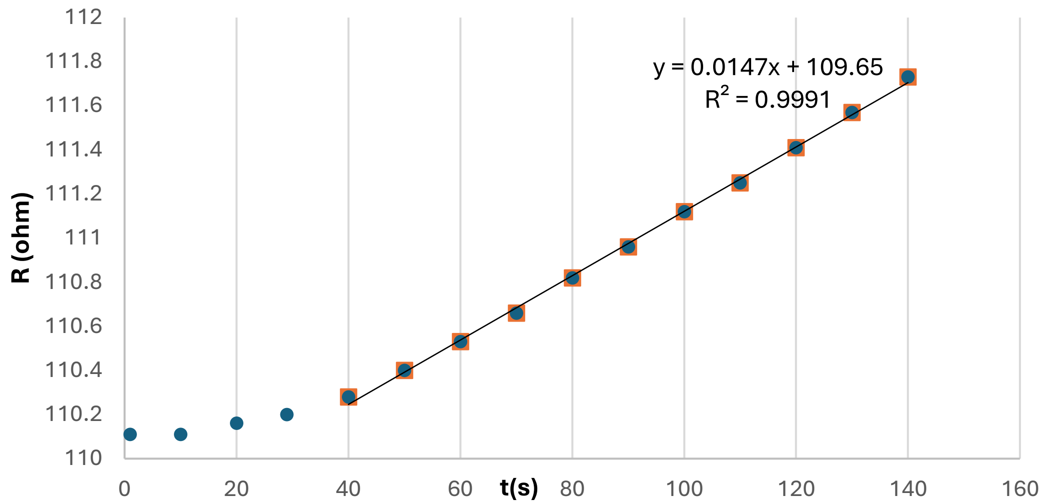
$$\theta_{en} = \frac{R - R_0}{R_0 \alpha}$$

$\theta_{en} = 25.87 \pm 0.03\ ℃$

A.2 (0.5 pt)

n	$R(\Omega)$	$t(s)$
1	110.11	1
2	110.11	10
3	110.15	20
4	110.18	30
5	110.26	40
6	110.4	50
7	110.53	60
8	110.66	70
9	110.82	80
10	110.96	90
11	111.12	100
12	111.25	110
13	111.41	120
14	111.57	130
15	111.73	140

A.3 (0.8 pt)

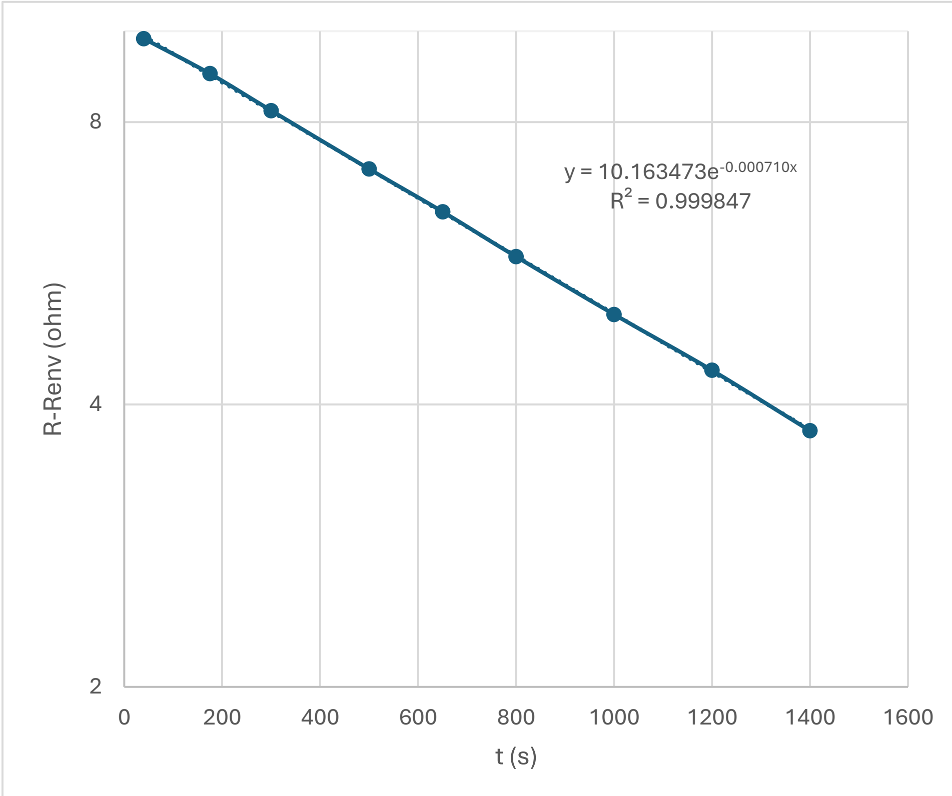


$\Delta R/\Delta t$	$0.0147\ \Omega/s$	$\frac{\Delta\theta}{\Delta t} = \frac{1}{R_0\alpha}\frac{\Delta R}{\Delta t}$ $C_s = \frac{P_1}{\frac{\Delta\theta}{\Delta t}}$, $\Delta C_s \approx C_s\frac{\Delta P_1}{P_1}$
$\Delta\theta/\Delta t$	0.03761 °C$/s$	
C_s	52 ± 2 J/°C	

A.4 (0.5pt)

n	$R(\Omega)$	$t(s)$	$(R - R_{en})(\Omega)$
1	119.98	40	9.82
2	119.17	175	9.01
3	118.39	300	8.23
4	117.29	500	7.13
5	116.58	650	6.42
6	115.91	800	5.75
7	115.15	1000	4.99
8	114.51	1200	4.35
9	113.91	1400	3.75
10	113.40	1600	3.24

A.5 (0.7 pt)

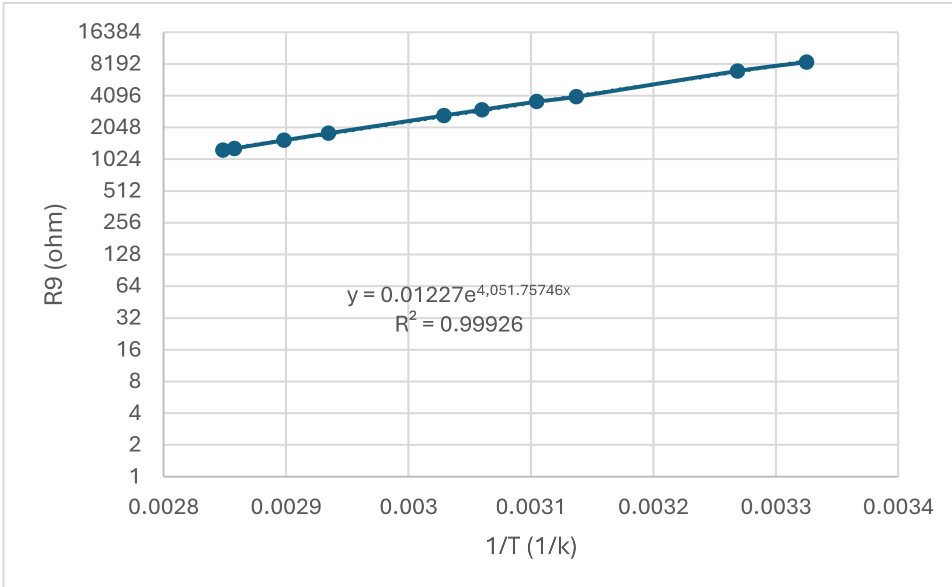


$\gamma = (710 \pm 3) \times 10^{-6}\ s^{-1}$

A.6 (0.5 pt)

n	$R(\Omega)$	$R_9(\Omega)$	$T(k)$	$\frac{1}{T}\left(\frac{1}{k}\right)$
1	8528	110.78	300.733	0.003325
2	7020	112.81	305.9272	0.003269
3	4005	117.83	318.772	0.003137
4	3601	119.13	322.0984	0.003105
5	3014	120.96	326.7808	0.00306
6	2658	122.27	330.1328	0.003029
7	1803	126.42	340.7515	0.002935
8	1547	128.09	345.0245	0.002898
9	1296	130.00	349.9117	0.002858
10	1246	130.45	351.0631	0.002848

A.7 (0.7 pt)



$E_g = 0.698 \pm 0.007\ eV$

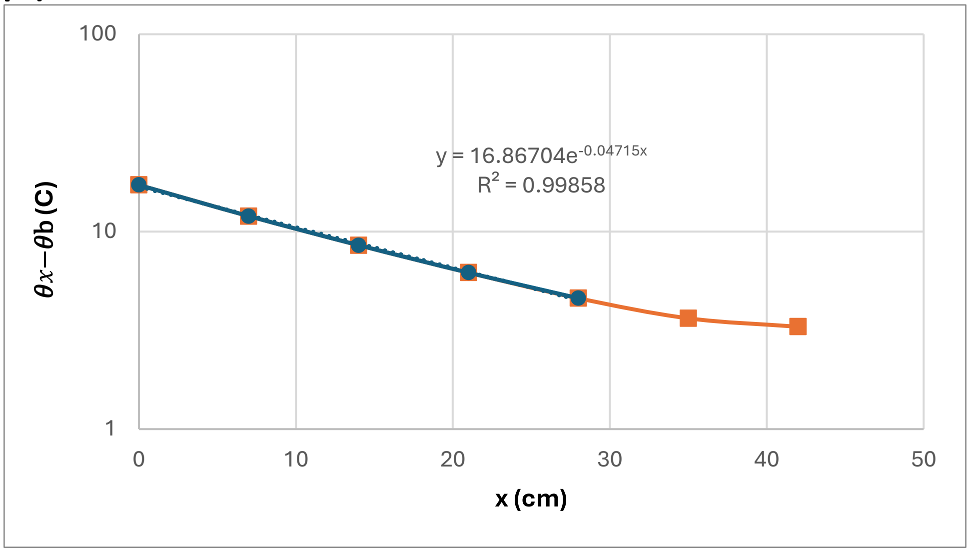
Part B: The long copper rod (4.1 points)

B.1 and B.5

$\theta_b = 24.41°C$

B.1 (0.4pt)				B.5 (0.4pt)	
n	$x(cm)$	$\theta_x(°C)$	$\theta_x - \theta_b(°C)$	$B^{(1)}e^{\lambda^{(0)}x}(°C)$	$\theta'_x - \theta_b(°C)$
1	0	44.61	17.23	0.27	16.96
2	7	39.37	11.99	0.37	11.62
3	14	35.92	8.54	0.51	8.03
4	21	33.58	6.20	0.72	5.48
5	28	31.98	4.60	1.00	3.60
6	35	31.02	3.64	1.39	2.25
7	42	30.68	3.30	1.93	1.37

B.2 (0.4 pt)



B.3 (0.6 pt)

$A^{(0)} = 16.87\ ℃$

$\lambda^{(0)} = 0.047 \pm\ 0.005 \quad \left(\frac{1}{cm}\right)$

B.4 (0.4 pt)

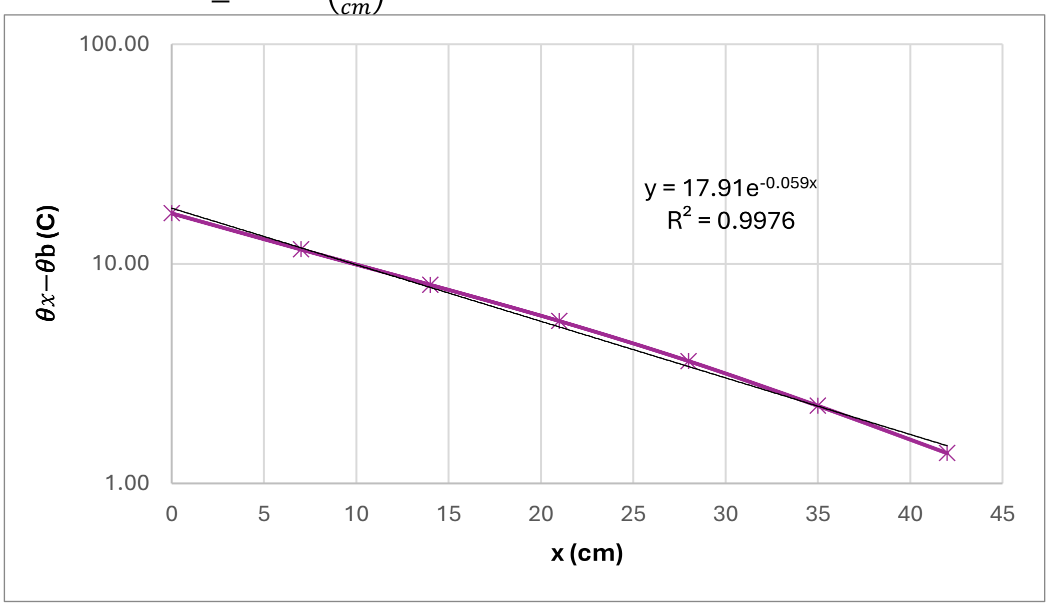
$B = Ae^{-2\lambda d}$

$B^{(1)} = 0.266℃$

B.6 (1.0 pt)

$A^{(1)} = 17.91℃$

$\lambda^{(1)} = 0.059 \pm\ 0.002 \left(\frac{1}{cm}\right)$



B.7 (0.9 pt)

$$P_2 = \int_{-0.5cm}^{42.5cm} 2\pi r h(\theta_x - \theta_b)dx \quad \text{or} \quad \int_0^d 2\pi r h(\theta_x - \theta_b)dx = \frac{2\pi r h A}{\lambda}(1 - e^{-2\lambda d})$$

$$h = \frac{P_2 \lambda}{2\pi r A(1 - e^{-2\lambda d})}, \quad k = \frac{2h}{\lambda^2 r}, P_2 = 4.5\ W$$

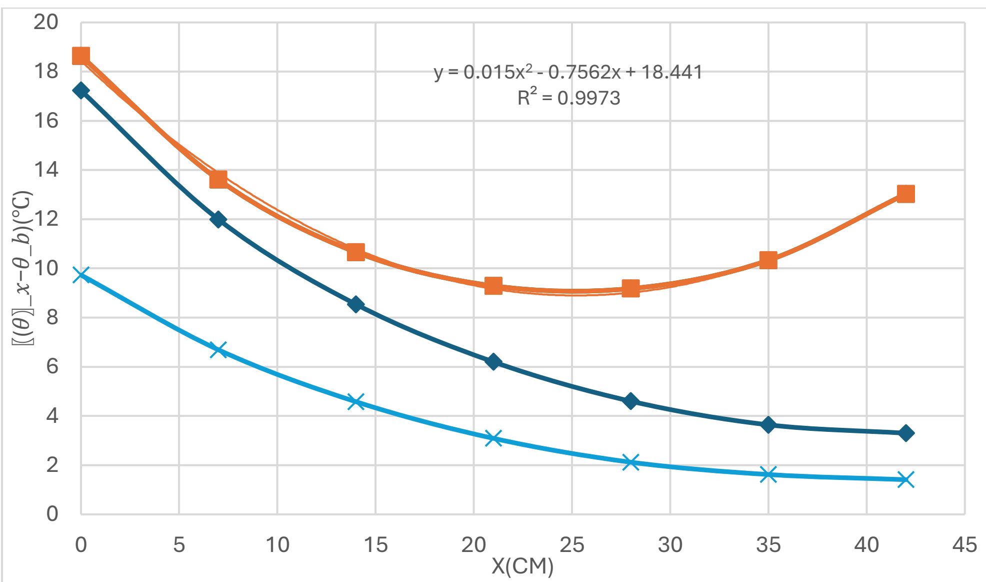
$$\lambda = 0.053 \pm 0.008 \left(\frac{1}{cm}\right), \ h = 0.0037 \pm 0.0003 \frac{W}{k\ cm^2}, k = 3.9 \pm 0.3 \frac{W}{k\ cm}$$

C.1 (0.4 pt)

$\theta_b = 27.38℃$

n	$x(cm)$	θ_x(℃)	$(\theta_x - \theta_b)$(℃)	B-1	C-B	Reversed order
1	0	46.02	18.64	17.23	1.41	9.73
2	7	40.99	13.61	11.99	1.62	6.69
3	14	38.04	10.66	8.54	2.12	4.58
4	21	36.67	9.29	6.2	3.09	3.09
5	28	36.56	9.18	4.6	4.58	2.12
6	35	37.71	10.33	3.64	6.69	1.62
7	42	40.41	13.03	3.3	9.73	1.41

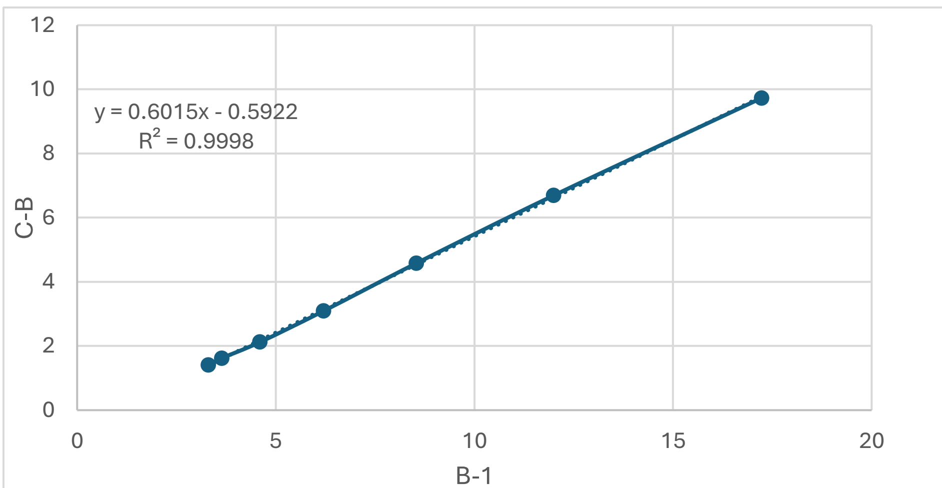
C.2 (0.6 pt)



(The blue plots are not necessary to draw)

C.3 (1.0 pt)

1) $\frac{P_3}{P_2} = \frac{\frac{d\theta}{dx}_{x=42cm}}{\frac{d\theta}{dx}_{x=0cm}} = \frac{\frac{\Delta\theta}{\Delta x}_{x=42cm}}{\frac{\Delta\theta}{\Delta x}_{x=0cm}} = \frac{\theta_7 - \theta_6}{\theta_0 - \theta_1}$
2) $\frac{P_3}{P_2} = \frac{\frac{d\theta}{dx}_{x=42cm}}{\frac{d\theta}{dx}_{x=0cm}} = \frac{\sinh(\lambda(42 - x_0))}{\sinh(\lambda x_0)}$
3) calculation by integral method or sigma in several parts
4) Slope ((C-B) _ B)



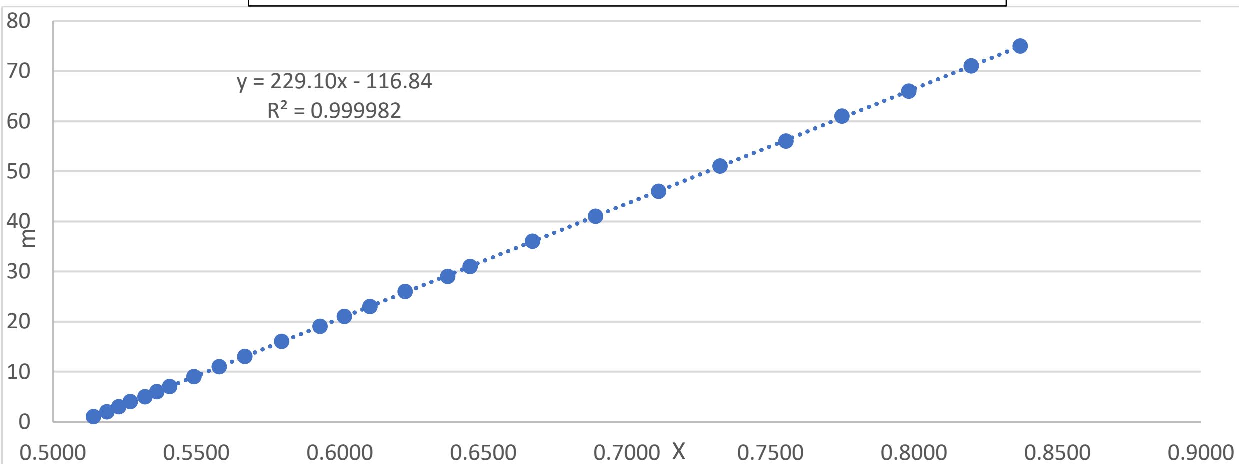
$$P_3 = 0.60\, P_2 \pm 0.01\, P_2$$

Problem E2- Solution

A.1			
number	m	θ_m (degrees)	$\sqrt{n^2 - \sin^2\theta_m} - \cos\theta_m$
1	1	9.00	0.5142
2	2	13.00	0.5188
3	3	15.75	0.5229
4	4	18.00	0.5270
5	5	20.50	0.5322
6	6	22.25	0.5362
7	7	24.00	0.5406
8	9	27.00	0.5491
9	11	29.75	0.5579
10	13	32.25	0.5668
11	16	35.50	0.5798
12	19	38.50	0.5931
13	21	40.25	0.6015
14	23	42.00	0.6105
15	26	44.25	0.6228
16	29	46.75	0.6375
17	31	48.00	0.6453
18	36	51.25	0.6671
19	41	54.25	0.6891
20	46	57.00	0.7110
21	51	59.50	0.7325
22	56	62.00	0.7555
23	61	64.00	0.7750
24	66	66.25	0.7982
25	71	68.25	0.8200
26	75	69.75	0.8371

S2-2

A.2



A.3

B = 229.1

A = -116.8

A.4

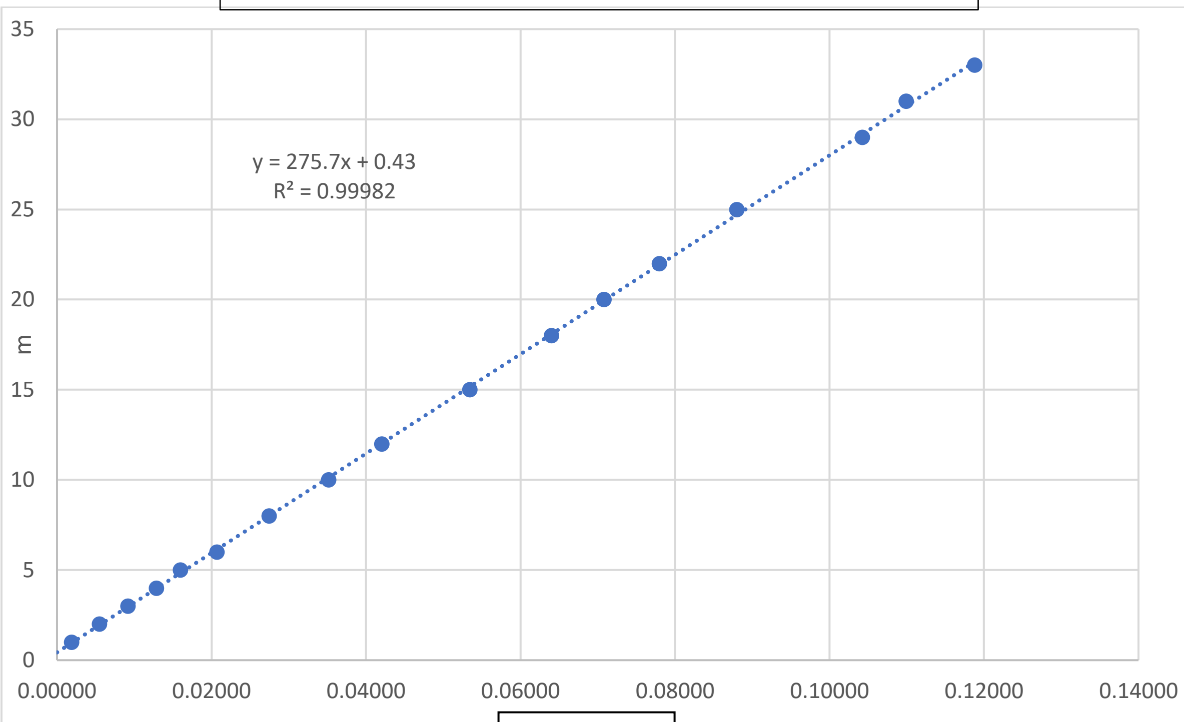
h = $(148.9 \pm 0.1)\ \mu m$

B.1			
number	m	θ_m (degrees)	${\theta_m}^2$ (rad^2)
1	1	2.50	0.00190
2	2	4.25	0.00550
3	3	5.50	0.00921
4	4	6.50	0.01287
5	5	7.25	0.01601
6	6	8.25	0.02073
7	8	9.50	0.02749
8	10	10.75	0.03520
9	12	11.75	0.04206
10	15	13.25	0.05348
11	18	14.50	0.06405
12	20	15.25	0.07084
13	22	16.00	0.07798
14	25	17.00	0.08803
15	29	18.50	0.10426
16	31	19.00	0.10997
17	33	19.75	0.11882

B.2

$$m = \frac{H}{2\lambda}\left(1 - \frac{1}{n}\right){\theta_m}^2$$

B.3



$\theta_m^2\ (rad^2)$

B.4

B = 275.7

A =0.43

B.5

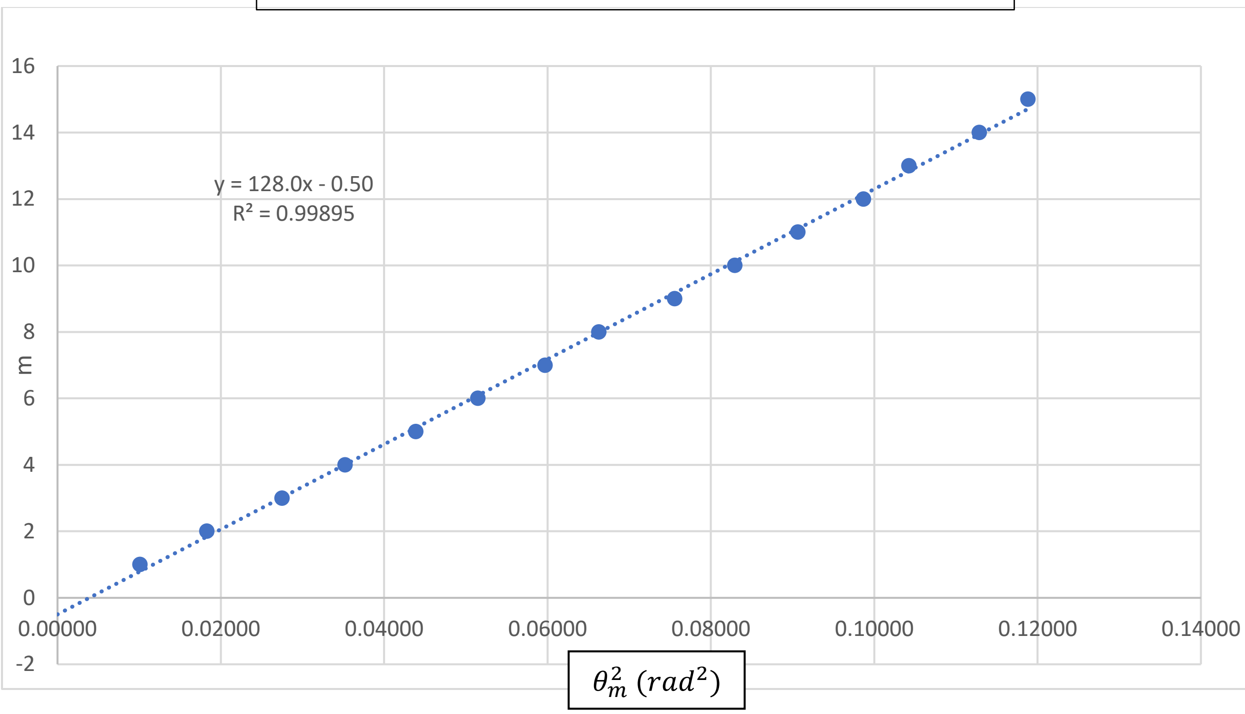
H = $(1.061 \pm 0.004)mm$

C.1			
number	m	θ_m (degree)	${\theta_m}^2$ (rad^2)
1	1	5.75	0.01007
2	2	7.75	0.01830
3	3	9.50	0.02749
4	4	10.75	0.03520
5	5	12.00	0.04386
6	6	13.00	0.05148
7	7	14.00	0.05971
8	8	14.75	0.06627
9	9	15.75	0.07556
10	10	16.50	0.08293
11	11	17.25	0.09064
12	12	18.00	0.09870
13	13	18.50	0.10426
14	14	19.25	0.11288
15	15	19.75	0.11882

C.2

$$m = \frac{H}{2\lambda}\left(N - \frac{N^2}{n}\right){\theta_m}^2$$

C.3



C.4

B = 128.0

A = -0.50

C.5

N =1.332 ± 0.002

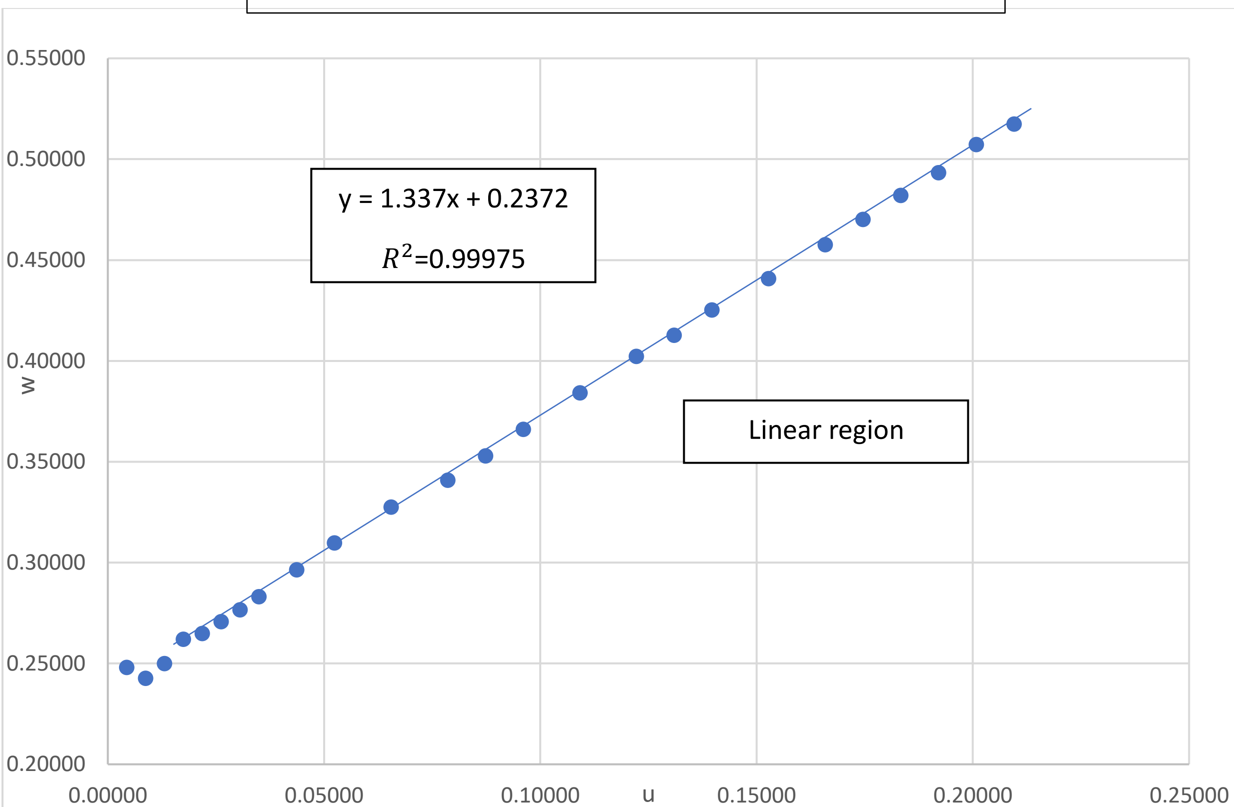
D.1			D.3	
number	m	θ_m (degree)	u	w
1	1	13.25	0.00436	0.24795
2	2	19.00	0.00873	0.24265
3	3	23.00	0.01309	0.24981
4	4	26.00	0.01746	0.26200
5	5	29.00	0.02182	0.26474
6	6	31.50	0.02619	0.27069
7	7	33.75	0.03055	0.27653
8	8	35.75	0.03492	0.28307
9	10	39.25	0.04365	0.29637
10	12	42.25	0.05238	0.30973
11	15	46.25	0.06547	0.32743
12	18	50.00	0.07857	0.34076
13	20	52.00	0.08730	0.35289
14	22	53.75	0.09603	0.36608
15	25	56.25	0.10912	0.38415
16	28	58.50	0.12222	0.40213
17	30	60.00	0.13095	0.41261
18	32	61.25	0.13968	0.42517
19	35	63.25	0.15277	0.44072
20	38	65.00	0.16587	0.45761
21	40	66.00	0.17460	0.47008
22	42	67.00	0.18333	0.48193
23	44	68.00	0.19205	0.49320
24	46	68.75	0.20078	0.50715
25	48	69.75	0.20951	0.51739

D.2

$$u = \frac{m\lambda}{h}$$

$$w = \frac{\frac{m\lambda}{h}\left(n + \frac{m\lambda}{2h}\right)}{1 - \cos\theta_m}$$

D.4



By removing the first 3 points

D.5

B = 1.337

A = 0.2372

D.6

N_B = 1.337±0.005

N_A = 1.3319 ± 0.0005

Theoretical calculations:

B.2 & C.2:

$$m=\frac{H}{\lambda}\left(\sqrt{n^2-N^2sin^2\theta_m}-Ncos\theta_m\right)-\frac{H}{\lambda}(n-N)$$

$$\theta_m \ll 1: \qquad \sin\theta_m \approx \theta_m \quad ; \qquad \cos\theta_m \approx 1-\frac{\theta_m^2}{2}$$

$$m=\frac{H}{\lambda}\left(n\sqrt{1-\frac{N^2\theta_m^2}{n^2}}-N\left(1-\frac{\theta_m^2}{2}\right)\right)-\frac{H}{\lambda}(n-N)$$

$$m=\frac{H}{\lambda}\left(n\left(1-\frac{N^2\theta_m^2}{2n^2}\right)-N\left(1-\frac{\theta_m^2}{2}\right)\right)-\frac{H}{\lambda}(n-N)$$

$$m=\frac{H}{2\lambda}N\left(1-\frac{N}{n}\right)\theta_m^2$$

$$N=1: \ m=\frac{H}{2\lambda}\left(1-\frac{1}{n}\right)\theta_m^2$$

D.2:

$$m=\frac{h}{\lambda}\left(\sqrt{n^2-N^2sin^2\theta_m}-Ncos\theta_m\right)-\frac{h}{\lambda}(n-N)$$

$$\frac{m\lambda}{h}+n-N(1-cos\theta_m)=\sqrt{n^2-N^2sin^2\theta_m}$$

$$\left(\frac{m\lambda}{h}\right)^2+2n\left(\frac{m\lambda}{h}\right)+n^2+N^2(1-cos\theta_m)^2-2N(1-cos\theta_m)\left(\frac{m\lambda}{h}+n\right)=n^2-N^2sin^2\theta_m$$

$$\left(\frac{m\lambda}{h}\right)^2+2n\left(\frac{m\lambda}{h}\right)+N^2(2-2cos\theta_m)-2Nn(1-cos\theta_m)-2N(1-cos\theta_m)\left(\frac{m\lambda}{h}\right)=0$$

$$\frac{\frac{m\lambda}{h}\left(n+\frac{m\lambda}{2h}\right)}{1-\cos\theta_m}+N^2-Nn-N\left(\frac{m\lambda}{h}\right)=0$$

$$\frac{\frac{m\lambda}{h}\left(n+\frac{m\lambda}{2h}\right)}{1-\cos\theta_m}=N(n-N)+\left(\frac{m\lambda}{h}\right)N$$

$$w=N(n-N)+uN$$

Error calculations:

Linear equation slope and intercept uncertainties:

$$\Delta B = B\sqrt{\frac{1}{n-2}\left(\frac{1}{r^2}-1\right)} \qquad ; \qquad \Delta A = \Delta B\sqrt{\bar{x}^2+\sigma_x^2}$$

C.5:

$$B = \frac{H}{2\lambda}\left(N-\frac{N^2}{n}\right) \Rightarrow \left(N-\frac{N^2}{n}\right) = \frac{2\lambda}{H}B \equiv c$$

$$\Rightarrow \frac{\Delta c}{c} = \sqrt{\left(\frac{\Delta B}{B}\right)^2+\left(\frac{\Delta H}{H}\right)^2}$$

$$\left(N-\frac{N^2}{n}\right) = c \Rightarrow N = \frac{n}{2} \pm \sqrt{\left(\frac{n}{2}\right)^2 - c\,n}$$

A negative sign is unacceptable in this equation.

$$N = \frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 - c\,n} \Rightarrow \Delta N = \frac{n}{2\sqrt{\left(\frac{n}{2}\right)^2 - c\,n}}\Delta c$$

D.6:

$$N_A(n-N_A) = A \Rightarrow N_A = \frac{n}{2} \pm \sqrt{\left(\frac{n}{2}\right)^2 - A}$$

A negative sign is unacceptable in this equation.

$$N_A = \frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 - A} \Rightarrow \Delta N_A = \frac{\Delta A}{2\sqrt{\left(\frac{n}{2}\right)^2 - A}}$$

Of course, this is calculated by ignoring the error of u and w. Since there is an h value in u and w, the h error causes an error in this quantity that is not included.