Brazilian Online Olympiad of Experimental Physics

1st Phase - September 20th and 21st, 2025

Name: ____________________________

Grade: ____________________________

Jayme Tiomno
Open Level
English

Exam Instructions

I. This exam contains **20** questions.

II. Each question has 5 answer choices, and only one is correct.

III. The maximum duration of this exam is **four hours**. In addition to the exam time, **5 minutes** will be given for filling out the answer sheet online.

IV. The use of calculators is permitted.

V. The exam must be taken individually, and discussing the solutions to the questions is not allowed during the exam period on **September 20th and 21st, 2025**.

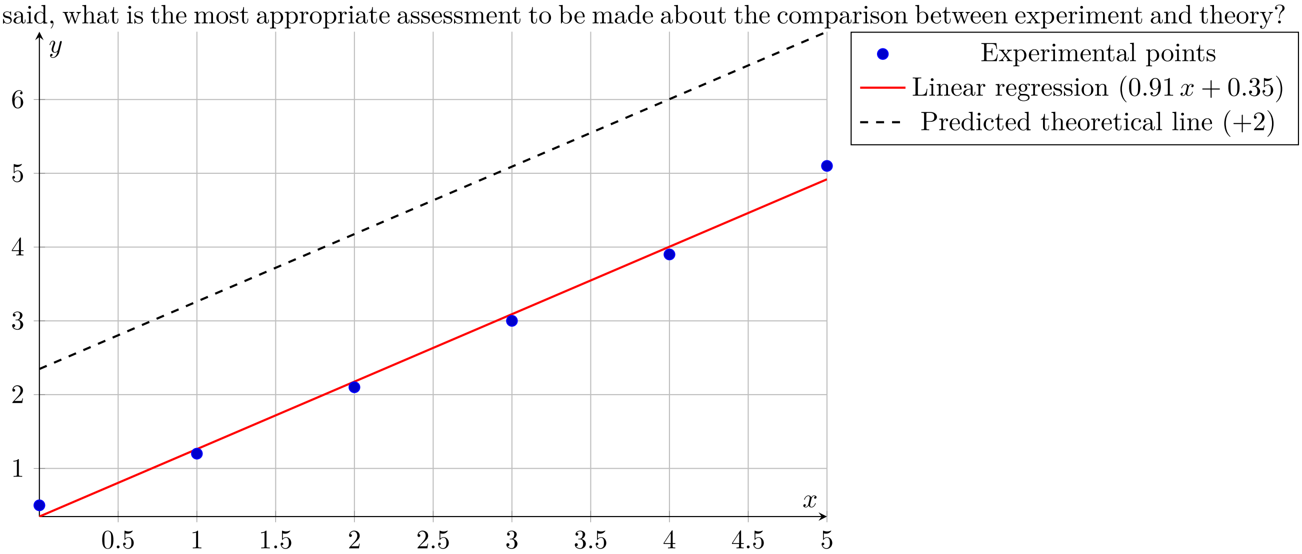
VI. If necessary, and unless otherwise stated, use: acceleration of gravity at the Earth's surface $g = 10\ \text{m/s}^2$; specific heat of liquid water $c_w = 1\ \text{cal/(g °C)}$; latent heat of fusion of ice $L = 80\ \text{cal/g}$; $1\ \text{cal} = 4.2\ \text{J}$; density of liquid water $\rho = 1.0\ \text{g/cm}^3$.

Support:

Trivia:

Jayme Tiomno (Rio de Janeiro, April 16, 1920 – Rio de Janeiro, January 12, 2011) was a prominent Brazilian theoretical physicist, internationally recognized for his contributions to particle physics. He was one of the founders of the Brazilian Center for Physics Research (CBPF) and a member of the Brazilian Academy of Sciences. His work on weak interactions, carried out in collaboration with giants like John Wheeler, was pioneering and led to the proposition of the universality of the weak interaction, a fundamental concept of the Standard Model. He received the Grand Cross of the National Order of Scientific Merit and is remembered for his decisive role in training new generations of physicists and in consolidating theoretical physics in Brazil.

Question 1. While conducting an experiment, Mary decided to record her data on a graph and compare it with the theoretically predicted line. Here is a picture of Mary's graph along with the theoretical line. That being said, what is the most appropriate assessment to be made about the comparison between experiment and theory?



a) It seems the experiment suffers mainly from random error, since the regression line is far from the theoretical line, despite the consistent distribution of the points (generating a low standard deviation).

b) It seems the experiment suffers mainly from systematic error, since the regression line is far from the theoretical line, despite the consistent distribution of the points (generating a low standard deviation).

c) It seems the experiment suffers mainly from random error, since the data distribution is very inconsistent (generating a high standard deviation).

d) It seems the experiment suffers mainly from systematic error, since the data distribution is very inconsistent (generating a high standard deviation).

e) The comparison between theory and experiment shows that they agree perfectly, with no relevant form of error.

Solution: Let's analyze the features of the graph:

- **Distribution of points:** The experimental points (blue) are very close to their own regression line (red). This indicates that the measurements were consistent with each other, with little scatter. Therefore, the **random error is low**.
- **Comparison with theory:** The regression line (red) is systematically offset from the theoretical

line (dashed black). They have similar slopes, but the experimental line is consistently below the theoretical one. This type of constant deviation is the definition of a **systematic error**.

Alternative (b) perfectly describes this situation: a systematic error (which moves the experimental line away from the theoretical one) and a consistent distribution (low random error). **Item B**

Text for questions 2 and 3: Jabriel Gemétrio conducted an experiment with an RC circuit and recorded his data in a table. Unfortunately, Jabriel is very clumsy and ends up spilling his black squid ink, which he kept at home, on some of the data in the table. For ease, these erased data points were labeled with letters and are in the following table:

Hint: It is known that, in an RC circuit: $V = V_0 e^{-t/\tau}$, where $\tau = RC$.

Table 1: Voltage measurements $V(t)$ over time with uncertainties.

Measurement	**Time $t \pm \Delta t$ (s)**	**Voltage $V \pm \Delta V$ (V)**
1	0.00 ± 0.05	5.00 ± 0.05
2	0.50 ± 0.05	3.80 ± 0.05
3	1.00 ± 0.05	$A \pm 0.05$
4	1.50 ± 0.05	2.20 ± 0.05
5	2.00 ± 0.05	1.65 ± 0.05
6	2.50 ± 0.05	1.25 ± 0.05
7	3.00 ± 0.05	0.95 ± 0.05
8	3.5 ± 0.05	0.72 ± 0.05
9	4.00 ± 0.05	$B \pm 0.05$
10	4.50 ± 0.05	0.42 ± 0.05

Question 2. By means of a regression with the plotted data, determine the approximate values of τ and V_0.

a) $\tau = 1.21$ s and $V_0 = 4.99\ V$

b) $\tau = 1.41$ s and $V_0 = 4.9\ V$

c) $\tau = 1.00$ s and $V_0 = 4.99\ V$

d) $\tau = 1.21$ s and $V_0 = 4.9\ V$

e) $\tau = 1.81$ s and $V_0 = 4.99\ V$

Solution: We can linearize the function by using logarithms:

$$\ln(V) = \ln(V_0) - \frac{t}{\tau}$$

Rearranging this into the form $y = a + bx$, we let $y = \ln(V)$ and $x = t$. The slope will be $b = -1/\tau$ and the y-intercept will be $a = \ln(V_0)$. By performing a linear regression on the available $(t, \ln(V))$ data points, we can find the coefficients of the line. From them:

$$\tau = -1/b \approx 1.81 \text{ s}$$

$$V_0 = e^a \approx 4.99 \text{ V}$$

Item E

Question 3. Find the expected values for data A and B, using your answer from the previous item.

a) $A \approx 2.18\ V$ and $B \approx 0.18\ V$

b) $A \approx 2.41\ V$ and $B \approx 0.29\ V$

c) $A \approx 1.84\ V$ and $B \approx 0.09\ V$

d) $A \approx 2.14\ V$ and $B \approx 0.18\ V$

e) $A \approx 2.88\ V$ and $B \approx 0.55\ V$

Solution: Now that we have the values for τ and V_0, we can use the RC circuit equation:

$$V(t) \approx 4.99 \cdot e^{-t/1.81}\ \text{V}$$

For data point A, $t = 1.00$ s:

$$A = V(1.00) \approx 4.99 \cdot e^{-1.00/1.81} \approx 2.88\ \text{V}$$

For data point B, $t = 4.00$ s:

$$B = V(4.00) \approx 4.99 \cdot e^{-4.00/1.81} \approx 0.55\ \text{V}$$

Item E

Question 4. Esmeralda is playing an electronic game about capturing little monsters. One day, she captures a Bulcaroma for the first time, which had a height of $h_0 = 2.33$ m. She believes it is rare for having a height greater than the normal for that species. So, to check if she is right, she decides to capture several other monsters of the same species and record their heights in the table. Esmeralda considers her original Bulcaroma rare if its height is $h_0 > h_{mean} + \sigma_h$, where h_{mean} is the average of the heights of all the monsters of that species she captured and σ_h is the sample standard deviation considering all heights. That being said, select the correct alternative.

Table: Heights recorded by Esmeralda

h(m)
2.33
1.98
1.95
1.90
2.20
2.15
2.50
2.24
2.30
2.32
2.21

a) The original Bulcaroma is rare, since $h_0 \approx h_{mean} + \sigma_h + 0.04$.

b) The original Bulcaroma is rare, since $h_0 \approx h_{mean} + \sigma_h + 0.08$.

c) The original Bulcaroma is not rare, since $h_0 \approx h_{mean} + \sigma_h - 0.04$.

d) The original Bulcaroma is not rare, since $h_0 \approx h_{mean} + \sigma_h - 0.08$.

e) The original Bulcaroma is not rare, since $h_0 \approx h_{mean} + \sigma_h$.

Solution: First, we calculate the mean (h_{mean}) of all 11 heights:

$$h_{\text{mean}} = \frac{2.33 + 1.98 + 1.95 + 1.90 + 2.20 + 2.15 + 2.50 + 2.24 + 2.30 + 2.32 + 2.21}{11} \approx 2.19 \text{ m}$$

Next, the sample standard deviation (σ_h):

$$\sigma_h = \sqrt{\frac{\sum(h_i - h_{\text{mean}})^2}{n-1}} \approx 0.18 \text{ m}$$

Now, we check the condition for rarity: $h_0 > h_{\text{mean}} + \sigma_h$.

$$h_{\text{mean}} + \sigma_h \approx 2.19 + 0.18 = 2.37 \text{ m}$$

Since $h_0 = 2.33$ m, the condition $2.33 > 2.37$ is false. The Bulcaroma is not rare. The difference is:

$$h_0 - (h_{\text{mean}} + \sigma_h) \approx 2.33 - 2.37 = -0.04 \text{ m}$$

Therefore, $h_0 \approx h_{\text{mean}} + \sigma_h - 0.04$.

Item C

Read the following text for the next two questions: It is quite common in electronic games that a certain character needs a certain amount of "experience" (which we will call E) to go up a "level" (which we will call N). However, it is also common that the amount of experience needed to level up increases as the levels go by. Suppose a game where E depends on N, approximately, in the form $E = A2^{BN}$ (where E is the experience needed to reach level $N + 1$). In it, a young person records the following data:

Table: Values of E for each N

N	**E**
1	2
2	5
3	9
4	18
5	37
6	74
7	150
8	290
9	590
10	1170

Question 5. Determine the approximate values of the coefficients A and B:

a) $A = 1.11$ and $B = 1.01$

b) $A \approx 0.159$ and $B = 1.01$

c) $A = 1.50$ and $B = 1.50$

d) $A = 0.30$ and $B = 1.01$

e) $A = 1.23$ and $B = 1.01$

Solution: We can linearize the function by applying the base-2 logarithm to both sides:

$$\log_2(E) = \log_2(A \cdot 2^{BN}) = \log_2(A) + BN$$

This is an equation for a straight line, $y = a + bx$, where $y = \log_2(E)$, $x = N$, $a = \log_2(A)$ (the y-intercept), and $b = B$ (the slope). By performing a linear regression on the points $(N, \log_2(E))$, we obtain $a \approx 0.149$ and $b \approx 1.01$. Thus:

$$B = b \approx 1.01$$

$$\log_2(A) = a \implies A = 2^a \approx 2^{0.149} \approx 1.11$$

Item A

Question 6. Choose the option that most closely approximates the amount of experience required for the character to level up from level 11 to level 15. Use the values from your answer in the previous item.

a) 17400.
b) 27400.
c) 37400.
d) 47400.
e) 57400.

Solution: The experience required to go from level 11 to 15 is the sum of the experience needed to reach levels 12, 13, 14, and 15. The experience needed to reach level $N+1$ is given by $E(N) = A \cdot 2^{BN}$. Using $A = 1.11$ and $B = 1.01$:

$$E_{\text{total}} = E(11) + E(12) + E(13) + E(14)$$

$$E_{\text{total}} = 1.11 \cdot (2^{1.01 \cdot 11} + 2^{1.01 \cdot 12} + 2^{1.01 \cdot 13} + 2^{1.01 \cdot 14}) \approx 37400$$

Item C

Question 7. George decides to take measurements of a quantity that follows the function $y = A^{Bx}$. However, when it came to making a graph of his measurements, he was unsure what he should do to create a linear graph. Therefore, select the correct alternative.

a) George can simply plot points (x, y) on a regular graph to obtain a linear graph.
b) George can plot points $(log(x), log(y))$ on a regular graph or plot the points (x,y) on a log-log graph.
c) George can plot points $(log(x), log(y))$ on a regular graph or plot the points (x,y) on a semi-log graph.
d) George can plot points $(x, log(y))$ on a regular graph or plot the points (x,y) on a log-log graph.
e) George can plot points $(x, log(y))$ on a regular graph or plot the points (x,y) on a semi-log graph.

Solution: To linearize the function $y = A^{Bx}$, we can apply a logarithm to both sides. Using the base-10 logarithm:

$$\log(y) = \log\left(A^{Bx}\right) = Bx \cdot \log(A)$$

Rearranging the terms:

$$\log(y) = (B \cdot \log(A)) \cdot x$$

This is an equation of the form $Y = mX$, where $Y = \log(y)$ and $X = x$. The relationship is linear between log(y) and x. Therefore, George should plot points $(x, \log(y))$ on a regular graph. A graph that uses a logarithmic scale for one axis (the y-axis, in this case) and a linear scale for the other (x-axis) is called a semi-log graph.

Item E

Read the following text for the next 3 questions: In a free-fall motion with air resistance, the acceleration of an object can be approximated as: $a = Ae^{-Bt}$. Curious, a young person records measurements of this motion in a table:

Table: Data of the falling motion

t (s)	a (m/s^2)
0.00	9.80
0.500	3.70
1.00	1.40
1.50	0.500
2.00	0.200
2.50	0.0700
3.00	0.0300

Question 8. Select the alternative with the approximate values of A and B.

a) $A = 8.68$ and $B = 0.95$

b) $A = 9.68$ and $B = 1.95$

c) $A = 10.7$ and $B = 2.95$

d) $A = 11.7$ and $B = 3.95$

e) $A = 12.7$ and $B = 4.95$

Solution: By linearizing the equation $a = Ae^{-Bt}$, we apply the natural logarithm:

$$\ln(a) = \ln(A) - Bt$$

This is an equation for a straight line $y = c - Bx$, where $y = \ln(a)$, $x = t$, and $c = \ln(A)$ is the y-intercept. Performing a linear regression on the data points $(t, \ln(a))$, we find a y-intercept $c \approx 2.27$ and a slope of approximately -1.95. Therefore:

$$B \approx -(-1.95) = 1.95 \text{ s}^{-1}$$

$$\ln(A) = c \approx 2.27 \implies A = e^{2.27} \approx 9.68 \text{ m/s}^2$$

Item B

Question 9. Record the approximate value of the uncertainty of B.

a) $\sigma_B = 0.5$

b) $\sigma_B = 0.2$

c) $\sigma_B = 0.02$

d) $\sigma_B = 0.09$

e) $\sigma_B = 0.9$

Solution: The uncertainty of the slope (b) from a linear regression can be estimated by the formula:

$$\sigma_b = |b|\sqrt{\frac{\frac{1}{r^2} - 1}{N - 2}}$$

Where b is the slope, r is the correlation coefficient, and N is the number of data points. Upon performing the linear regression, we find $b \approx -1.95$, $N = 7$, and a correlation coefficient r that is very close to -1 (e.g., $r^2 \approx 0.999$).

Substituting these values, we find $\sigma_b \approx 0.02$. Since $B = -b$, the uncertainty is the same: $\sigma_B = \sigma_b \approx 0.02$.

Item C

Question 10. According to the value of the correlation coefficient of the data, the young man evaluates the precision of the model as follows: Inaccurate: $r^2 \leq 0.97$
Slightly accurate: $0.97 < r^2 \leq 0.98$
Accurate: $0.98 < r^2 \leq 0.99$
Very accurate: $r^2 > 0.99$
Thus, how is the model classified according to the young man's model? Remember to use significant figures correctly.

a) Inaccurate.

b) Slightly accurate.

c) Accurate.

d) Very accurate.

e) It is impossible to know.

Solution: By performing a linear regression on the linearized data $(t, \ln(a))$, the coefficient of determination (r^2) is calculated. The resulting value is approximately $r^2 \approx 0.999$. According to the provided classification, since $0.999 > 0.99$, the model is classified as "Very accurate".

Item D

Read the following text for the next three questions: Totomelli records the following data on some quantities: $A = 0.40$; $B = 0.11$; $C = 0.234$; $D = 0.0532$. However, he wants to calculate the quantities: $E = A + B - C + D$; $F = \frac{B+D}{A+C}$.

Question 11. With how many significant figures will E and F be expressed? For this item, consider that all quantities have negligible uncertainties.

a) E: 2. F: 2.

b) E: 4. F: 4.

c) E: 1. F: 2.

d) E: 2. F: 4.

e) E: 1. F: 1.

Solution: To solve this, we apply the rules for significant figures, assuming that $A = 0.40$.

- **For E (Addition/Subtraction):** The result's precision is limited by the term with the fewest decimal places. Both $A = 0.40$ and $B = 0.11$ have two decimal places. Thus, the result for E must be rounded to two decimal places, which will result in a number with **2 significant figures** (of the form $0.xx$).
- **For F (Division):** The result has the same number of significant figures as the term with the fewest significant figures. We must first evaluate the numerator and denominator.
  - Numerator $(B + D)$: $0.11 + 0.0532$ is rounded to 2 decimal places, giving 0.16 (**2 significant figures**).
  - Denominator $(A + C)$: $0.40 + 0.234$ is rounded to 2 decimal places, giving 0.63 (**2 significant figures**).

  Since both terms in the division have 2 significant figures, the final result for F must also have **2 significant figures**.

Therefore, both E and F will be expressed with 2 significant figures. **Item A**

Question 12. For this item only, it is known that the uncertainties are $\sigma_A = 0.04$, $\sigma_B = 0.01$, $\sigma_C = 0.02$ and $\sigma_D = 0.005$, which are, respectively, the uncertainties of A, B, C, and D. Calculate the value of E along with its uncertainty.

a) 0.3292±0.065

b) 0.329±0.046

c) 0.33±0.05

d) 0.33±0.1

e) 0.33±0.07

Solution: First, we calculate the central value of E:

$$E = A + B - C + D = 0.4 + 0.11 - 0.234 + 0.0532 = 0.3292$$

For addition and subtraction, the uncertainty is found by adding the individual uncertainties in quadrature:

$$\sigma_E = \sqrt{\sigma_A^2 + \sigma_B^2 + \sigma_C^2 + \sigma_D^2} = \sqrt{(0.04)^2 + (0.01)^2 + (0.02)^2 + (0.005)^2}$$

$$\sigma_E = \sqrt{0.0016 + 0.0001 + 0.0004 + 0.000025} = \sqrt{0.002125} \approx 0.04609$$

The uncertainty is typically rounded to one or two significant figures. Rounding to one gives $\sigma_E \approx 0.05$. The central value of E must then be rounded to the same decimal place (the hundredths place).

$$E \approx 0.33$$

The final result is $E = 0.33 \pm 0.05$.

Item C

Question 13. For this item only, it is known that the uncertainty of F is $\sigma_F = 0.03$. Therefore, calculate the value of F along with its uncertainty.

a) 0.2667 ± 0.03

b) 0.2574 ± 0.03

c) 0.27 ± 0.03

d) 0.26 ± 0.03

e) 0.30 ± 0.03

Solution: First, we calculate the central value of F:

$$F = \frac{B+D}{A+C} = \frac{0.11 + 0.0532}{0.4 + 0.234} = \frac{0.1632}{0.634} \approx 0.257413...$$

The uncertainty is given as $\sigma_F = 0.03$. Since the uncertainty is given to the hundredths place, we must round the central value of F to the same decimal place.

$$F \approx 0.26$$

The final result is $F = 0.26 \pm 0.03$.

Item D

Question 14. A function depends on other variables in the form: $y = a^1 b^2 c^3$. Therefore, determine the uncertainty σ_y of y as a function of the variables a, b, and c and their uncertainties (σ_a, σ_b and σ_c).

a) $\sigma_y = ab^2c^3\sqrt{\frac{\sigma_a^2}{a^2} + \frac{\sigma_b^2}{b^2} + \frac{\sigma_c^2}{c^2}}$

b) $\sigma_y = 2ab^2c^3\sqrt{\frac{\sigma_a^2}{a^2} + \frac{\sigma_b^2}{b^2} + \frac{\sigma_c^2}{c^2}}$

c) $\sigma_y = 3ab^2c^3\sqrt{\frac{\sigma_a^2}{a^2} + \frac{\sigma_b^2}{b^2} + \frac{\sigma_c^2}{c^2}}$

d) $\sigma_y = ab^2c^3\sqrt{3\frac{\sigma_a^2}{a^2} + 2\frac{\sigma_b^2}{b^2} + \frac{\sigma_c^2}{c^2}}$

e) $\sigma_y = ab^2c^3\sqrt{\frac{\sigma_a^2}{a^2} + 4\frac{\sigma_b^2}{b^2} + 9\frac{\sigma_c^2}{c^2}}$

Solution: For functions involving products and powers, we use the rule for relative uncertainty propagation. The square of the relative uncertainty of the function is the sum of the squares of the relative uncertainties of each variable, multiplied by their respective exponents squared.

$$\left(\frac{\sigma_y}{y}\right)^2 = \left(1 \cdot \frac{\sigma_a}{a}\right)^2 + \left(2 \cdot \frac{\sigma_b}{b}\right)^2 + \left(3 \cdot \frac{\sigma_c}{c}\right)^2$$

$$\frac{\sigma_y^2}{y^2} = \frac{\sigma_a^2}{a^2} + 4\frac{\sigma_b^2}{b^2} + 9\frac{\sigma_c^2}{c^2}$$

Solving for σ_y:

$$\sigma_y = y\sqrt{\frac{\sigma_a^2}{a^2} + 4\frac{\sigma_b^2}{b^2} + 9\frac{\sigma_c^2}{c^2}}$$

Item E

Question 15. Doriton is tired of studying in his noisy room and, therefore, decided to prove that the noise was too loud by analyzing the sound intensity level in the location. The intensity level is given by: $N = 10\log\left(\frac{I}{I_0}\right)$ where $r = I/I_o$ is the ratio between sound intensities, compared to a reference value. After measurements, Doriton finds that the average value of r is 50.00, with an uncertainty of $\sigma_r = 2.000$. Therefore, determine the sound intensity level corresponding to the average r, along with its uncertainty.

a) 17.00 ± 0.13

b) 17.00 ± 0.17

c) 17.00 ± 0.13

d) 15.00 ± 0.13

e) 15.00 ± 0.15

Solution: First, we calculate the sound intensity level N for $r = 50$:

$$N = 10\log_{10}(50) \approx 10 \cdot 1.69897 \approx 16.99 \text{ dB} \approx 17.00 \text{ dB}$$

The uncertainty is found by error propagation: $\sigma_N = |\frac{dN}{dr}|\sigma_r$.

$$\frac{dN}{dr} = \frac{d}{dr}(10\log_{10}(r)) = 10\frac{1}{r\ln(10)}$$

$$\sigma_N = \left|\frac{10}{50\ln(10)}\right| \cdot 2.000 = \frac{20}{50\ln(10)} \approx \frac{0.4}{2.3026} \approx 0.17 \text{ dB}$$

Item B

Question 16. In an experiment, you are asked to raise a ramp of $l = 15.0$ cm (with negligible uncertainty) to an angle of $\theta = 30°$ with respect to the horizontal, resting it against a wall. To do this, you must only measure the leg corresponding to the horizontal with your ruler, which has an uncertainty of $\sigma_x = 0.5$ *mm*. Thus, determine the uncertainty of the angle, approximately.

a) $5 \cdot 10^{-3\circ}$

b) $3 \cdot 10^{-1\circ}$

c) $2 \cdot 10^{-3\circ}$

d) $1 \cdot 10^{-1\circ}$

e) $4 \cdot 10^{-3\circ}$

Solution: The relationship between the horizontal leg x, the hypotenuse l, and the angle θ is $x = l\cos(\theta)$. We can find the relationship between the uncertainties by differentiating the expression $\cos(\theta) = x/l$.

$$-\sin(\theta)d\theta = \frac{dx}{l}$$

For uncertainties, we use the absolute values:

$$\sigma_\theta \approx \left|\frac{1}{l\sin(\theta)}\right|\sigma_x$$

Converting units to cm: $l = 15.0$ cm, $\sigma_x = 0.05$ cm, $\theta = 30°$.

$$\sigma_\theta(\text{rad}) \approx \frac{0.05}{15.0 \cdot \sin(30°)} = \frac{0.05}{15.0 \cdot 0.5} = \frac{0.05}{7.5} \approx 0.00667 \text{ rad}$$

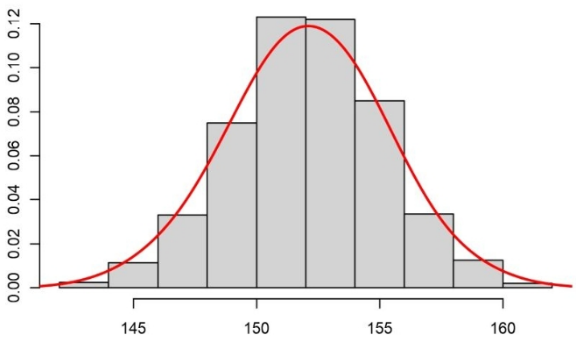
Converting radians to degrees:

$$\sigma_\theta(\text{degrees}) = 0.00667 \cdot \frac{180°}{\pi} \approx 0.38°$$

The closest value is 0.3°, which is $3 \cdot 10^{-1}$°.

Item B

Read the Text for the next questions: One day, Jabriel Mentos decided to do a curious experiment after seeing he had two non-biased dice at home. He throws the dice several times and notes the sum of the numbers on each die. Upon making a histogram of the results, he noticed that the result closely resembles that of a Gaussian curve:



Curious, Jabriel researches more about this curve and discovers that it is very important in the statistical study of certain systems. The peak of the curve signifies the event with the highest probability; in this case, the event that the sum of the dice is 7. However, the Gaussian curve is very interesting because it shows how the probability of events behaves as they move away from the peak. In a regime dominated by the Gaussian curve, we have the following formula for calculating the probability: $P(x) = Ce^{-x^2/2a^2}$. Let's study the Gaussian curve in the next items. It might be useful to use the integral: $I(b) = \int_{-\infty}^{+\infty} e^{-bx^2} dx = \sqrt{\frac{\pi}{b}}$.

Question 17. Knowing that the sum of probabilities is 1, calculate the value of the coefficient C.

a) $C = \frac{1}{\sqrt{a^2\pi}}$
b) $C = \frac{1}{\sqrt{2a^2\pi}}$
c) $C = \frac{2}{\sqrt{2a^2\pi}}$
d) $C = \frac{1}{\sqrt{4a^2\pi}}$
e) $C = \frac{2}{\sqrt{a^2\pi}}$

Solution: The sum of all probabilities must be 1. This means the integral of the probability function $P(x)$ from $-\infty$ to $+\infty$ must be 1.

$$\int_{-\infty}^{+\infty} P(x)dx = \int_{-\infty}^{+\infty} Ce^{-x^2/2a^2} dx = 1$$

We can take the constant C out of the integral:

$$C \int_{-\infty}^{+\infty} e^{-x^2/2a^2} dx = 1$$

The integral has the form $I(b)$ with $b = 1/(2a^2)$. Using the given formula:

$$\int_{-\infty}^{+\infty} e^{-x^2/2a^2} dx = \sqrt{\frac{\pi}{1/(2a^2)}} = \sqrt{2a^2\pi}$$

Substituting this back:

$$C\sqrt{2a^2\pi} = 1 \implies C = \frac{1}{\sqrt{2a^2\pi}}$$

Item B

Question 18. Calculate the mean value of x.

a) a

b) 2a

c) πa

d) $2\pi a$

e) 0

Solution: The mean value of x, denoted by $\langle x \rangle$, is calculated by:

$$\langle x \rangle = \int_{-\infty}^{+\infty} xP(x)dx = \int_{-\infty}^{+\infty} xCe^{-x^2/2a^2} dx$$

The integrand, $f(x) = xe^{-x^2/2a^2}$, is an odd function, because $f(-x) = (-x)e^{-(-x)^2/2a^2} = -xe^{-x^2/2a^2} = -f(x)$. The integral of an odd function over a symmetric interval (from $-\infty$ to $+\infty$) is always zero. Therefore, the mean value of x is 0.

Item E

Question 19. Calculate the mean square value of x. This is very important for analyzing the variance of the system (and thus, also the standard deviation). Hint: differentiate the given integral with respect to b.

a) $a^2/\sqrt{2}$

b) $a^2/2$

c) a^2

d) $\sqrt{2}a^2$

e) $2a^2$

Solution: The mean square value, $\langle x^2 \rangle$, is:

$$\langle x^2 \rangle = \int_{-\infty}^{+\infty} x^2P(x)dx = C\int_{-\infty}^{+\infty} x^2e^{-x^2/2a^2} dx$$

Following the hint, we differentiate $I(b) = \sqrt{\pi/b}$ with respect to b:

$$\frac{dI}{db} = \frac{d}{db}\left(\int_{-\infty}^{+\infty} e^{-bx^2} dx\right) = \int_{-\infty}^{+\infty} -x^2e^{-bx^2} dx$$

Also, $\frac{d}{db}(\sqrt{\pi}b^{-1/2}) = \sqrt{\pi}(-\frac{1}{2}b^{-3/2}) = -\frac{1}{2}\sqrt{\frac{\pi}{b^3}}$. Equating the two expressions gives $\int_{-\infty}^{+\infty} x^2e^{-bx^2} dx =$

$\frac{1}{2}\sqrt{\frac{\pi}{b^3}}$. For our integral, $b = 1/(2a^2)$ and $C = 1/\sqrt{2a^2\pi}$.

$$\langle x^2 \rangle = \frac{1}{\sqrt{2a^2\pi}}\left(\frac{1}{2}\sqrt{\frac{\pi}{(1/2a^2)^3}}\right) = \frac{1}{\sqrt{2a^2\pi}}\left(\frac{1}{2}\sqrt{8a^6\pi}\right) = \frac{1}{\sqrt{2a^2\pi}}\left(\frac{2a^3\sqrt{2\pi}}{2}\right) = a^2$$

Item C

Question 20. Luís was performing his experiment normally when he realized he had to calculate the uncertainty of a strange function. In his experiment, the function that relates the data is of the form: $y = \frac{(\ln(x))^3}{x}$. Therefore, select the alternative that correctly relates the uncertainty of $y(\sigma_y)$ with the uncertainty of $x(\sigma_x)$.

a) $\sigma_y = \frac{3\ ln(x)\sigma_x}{x^2}$
b) $\sigma_y = \frac{3\ ln(x)^2\sigma_x}{x^2}$
c) $\sigma_y = \frac{\sigma_x}{x^2}((\ln x)^3 + (\ln x)^2)$
d) $\sigma_y = \frac{\sigma_x}{x^2}(3(\ln x)^2 - (\ln x)^3)$
e) $\sigma_y = \frac{ln(x)^3\sigma_x}{x^2}$

Solution: To find the uncertainty σ_y, we use error propagation with derivatives, where $\sigma_y = \left|\frac{dy}{dx}\right|\sigma_x$. First, we must calculate the derivative of the function $y = \frac{(\ln(x))^3}{x}$ using the quotient rule $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$. Let $u = (\ln x)^3$ and $v = x$.

- The derivative of u is $u' = 3(\ln x)^2 \cdot \frac{1}{x}$ (by the chain rule).
- The derivative of v is $v' = 1$.

Substituting into the quotient rule:

$$\frac{dy}{dx} = \frac{\left(3(\ln x)^2 \cdot \frac{1}{x}\right) \cdot x - ((\ln x)^3 \cdot 1)}{x^2}$$

$$\frac{dy}{dx} = \frac{3(\ln x)^2 - (\ln x)^3}{x^2}$$

Therefore, the uncertainty is:

$$\sigma_y = \left|\frac{3(\ln x)^2 - (\ln x)^3}{x^2}\right|\sigma_x = \frac{\sigma_x}{x^2}|3(\ln x)^2 - (\ln x)^3|$$

Item D