Brazilian Online Physics Olympiad

2nd Phase - October 26th and 27th, 2025

Name: ____________________________

Grade: ____________________________

César Lattes
Open Level
English

Exam Instructions

I. This exam consists of **3** questions.

II. The maximum duration of this exam is **four hours**. In addition to the exam time, **5 minutes** will be given for filling out the online answer sheet.

III. The exam must be taken individually and discussing the solutions to the questions is not allowed during the exam period **October 26th and 27th, 2025**.

IV. If necessary, and unless indicated otherwise, use: gravitational acceleration on Earth's surface $g = 10\,\mathrm{m/s^2}$; specific heat of liquid water $c_a = 1\,\mathrm{cal/(g\,^\circ C)}$; latent heat of fusion of ice $L = 80\,\mathrm{cal/g}$; $1\,\mathrm{cal} = 4.2\,\mathrm{J}$; density of liquid water $\rho = 1.0\,\mathrm{g/cm^3}$; Wien's constant $b = 2.898 \times 10^{-3}$ m $\cdot$ K; Planck's constant $h = 6.63 \times 10^{-34}$ J $\cdot$ s; speed of light $c = 3 \times 10^8$ m/s; energy of hydrogen in the ground state $E_n = -\dfrac{13.6}{n^2}$ eV; 1 eV $= 1.6 \times 10^{-19}$ J.

Support:

Trivia:

Cesare Mansueto Giulio Lattes, better known as César Lattes (Curitiba, July 11, 1924 — Campinas, March 8, 2005), was a Brazilian physicist, co-discoverer of the pion (π-meson), a discovery that led to the 1950 Nobel Prize in Physics being awarded to Cecil Frank Powell, the research leader. Lattes is one of Brazil's most distinguished physicists, and his work was fundamental to the development of atomic physics in the country. He was also a great leader in the Brazilian scientific community and one of the main figures responsible for the creation of the National Council for Scientific and Technological Development (CNPq).

Question 1: Thermal Shield - 10 points[1]

A future mission to Mars plans to use a new engine that requires its fuel to be kept at an extremely low temperature, below its critical boiling point, T_c. To prevent the fuel from evaporating, the spherical tank storing it is surrounded by a thermal insulation system composed of N thin, concentric spherical shields, separated by vacuum. To simplify the analysis, consider that the areas of all shields and the tank are approximately equal to A. The outermost shield is exposed to a constant solar energy flux F_0.
The tank itself generates a small amount of heat, P_{int}, due to monitoring equipment. The total thermal load (heat from outside plus internally generated) is removed by an active cryocooler, which has a maximum refrigeration capacity of $P_{\max}$. The insulation design must be efficient enough so that the total thermal load does not exceed this capacity. For the entire analysis, consider the system in thermal equilibrium, the temperature of deep space as zero, and the Stefan-Boltzmann constant as σ.

Part A: The Ideal Model (Black Bodies) - 4 points

In this part, as a first approximation, we will model the system as if all surfaces (the tank and the N shields) were **perfect black bodies**, with emissivity and absorptivity equal to 1.

(a) **0.6 points** In equilibrium, a constant net heat flux, Q_{net}, passes through each vacuum layer. Write an expression for this flux between shield i (at temperature T_i) and the adjacent shield $i+1$ (at temperature T_{i+1}).

(b) **1.4 points** Show that the relationship between the temperature of the first shield (T_1) and the tank (T_{tanque}) can be written as a function of N and Q_{net}.

(c) **2 points** Use the energy balance on the first shield (the outer boundary condition) and the thermal load constraint on the tank (the inner boundary condition) to find an expression for the minimum number of shields, N_{ideal}, required to maintain the tank at $T_{\text{tanque}} \leq T_c$.

Part B: The Realistic Model (Gray Surfaces) - 6 points

In reality, the materials used for insulation are highly reflective. Now consider that all surfaces have a constant emissivity ϵ much less than 1.

(d) **3 points** First, derive an expression for the net energy flux per unit time between two large parallel surfaces with the same area A and emissivity ϵ, maintained at temperatures T_1 and T_2.

(e) **2 points** Using the result from the previous item, find the new expression for the minimum number of shields, N_{real}, as a function of ϵ and the other variables of the problem.

[1] By Lucas Praça

(f) 1 point Compare your expressions for N_{real} and N_{ideal}. For a value of $\epsilon \ll 1$, which of the two numbers of shields is larger? Physically justify why the use of low-emissivity materials is such a crucial factor for thermal insulation design in space missions.

Question 2: Coffee Optics - 10 points[2]

When a cup of coffee is illuminated by a lamp or sunlight, it is common to notice bright curved shapes on the surface of the liquid or at the bottom of the cup. These shapes, known as **caustics**, result from the reflection or refraction of light rays, which concentrate in certain regions of space. This is a stunning everyday example of how the geometry of a curved surface can concentrate light, forming patterns that are both aesthetically striking and physically relevant. The study of caustics involves concepts from geometric optics, such as the law of reflection, and is related to classic problems that also appear in advanced optical and astronomical systems.

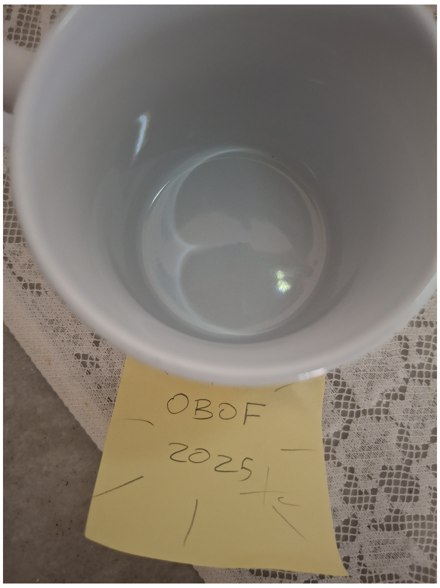

Figure 1: Caustic pattern in a coffee mug.

(A) 6 points Let C be a small convex mirror with focal length f, and let Γ be a circle with diameter d_1 tangent to C. Suppose P is a point light source located on Γ. Show that the locus of all images of P is another circle Ω, tangent to C, with diameter d_2 satisfying

$$\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f}.$$

(B) 4 points Let P be a fixed point light source and let Γ be a circular mirror of radius R. Show that

- if $P \in \Gamma$, the caustic is a **cardioid**;
- if P is at infinity, the caustic is half of a **nephroid**.

In both cases, these curves can be generated as the path traced by a point on the circumference of a circle that rolls without slipping around another fixed circle of a different radius. Determine the radii of these circles, sketch the curves, and explain why this rolling circle construction does not apply in the general case.

[2] By Paulo Vinícius

Question 3: Waves on a String - 10 points[3]

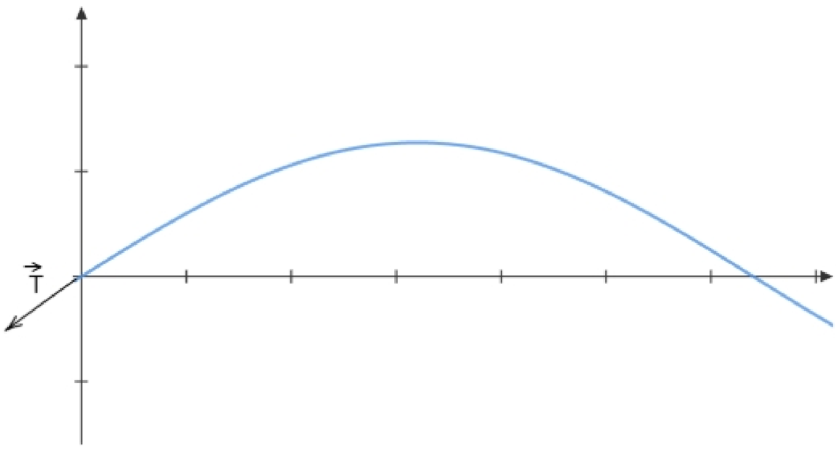

Figure 2: Initial configuration of the string under tension $\vec{T}$.

In the figure above, we have a string with linear mass density ρ which is set to oscillate by a tension T. This force will propagate energy to the string, which will exhibit interesting characteristics. Our intention is to start from a simple case and arrive at a better understanding of waves in general.

Part A: The Wave Equation - 3 points

(a) 2 points Set up the differential equation of motion for the system.

(b) 1 point Separate the equation into spatial and temporal parts [Hint: A function $f(x;t)$ can be written as: $\xi(x)\varphi(t)$].

Part B: Energy and Power - 4 points

We will use as a solution:

$$y(x;t) = \mathrm{Re}[Ae^{i(kx-\omega t)}] = A\cos\left(kx - \omega t + \frac{\pi}{2}\right)$$
$$y(0;0) = 0 \quad v(0;0) = A\omega$$

(c) 2 points Calculate the average power of the energy supplied to the string.

(d) 2 points Express the formulas (in integral form) for the total kinetic and potential energy of the system.

Part C: Principle of Least Action - 3 points

There is a quantity in physics that is useful from general relativity to quantum mechanics, and its name is action, which is defined as:

$$S = \int L(y;\dot{y};t)dt; \quad L = T - V$$

[3] By Yvens Amaral

However, when we have systems that, unlike particles, are dispersed in space like strings, we must use $\mathcal{L}$ which is defined as the density of L. We therefore have our new expression:

$$S = \int \mathcal{L}(y; \frac{\partial y}{\partial t}; \frac{\partial y}{\partial x}; x; t) dx dt$$

It is general knowledge that nature evolves to seek the most stable situations, which are those of minimum action. Let's then minimize the action. Given a deformation $y' = y + \epsilon\alpha(x)\beta(t)$, with $\alpha(x_1) = \alpha(x_2) = \beta(t_1) = \beta(t_2) = 0$, and minimizing the action by integration by parts, we arrive at the following equation (Euler-Lagrange Equation):

$$\frac{\partial \mathcal{L}}{\partial y} - \frac{\partial(\frac{\partial \mathcal{L}}{\partial y})}{\partial t} - \frac{\partial(\frac{\partial \mathcal{L}}{\partial y'})}{\partial x} = 0$$

(e) **3 points** Using the results from item (d) and the formula above, set up the equation of motion for the system and compare it with the one from item (a).